The Optimal Structure of Boards : A Theoretical Approach

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Abstract

This paper provides a theoretical rationale for the observed structure of boards. While the previous models focus on the strategic interaction among CEO, insiders, and outsiders, I assume that only the firm (or shareholder) makes a decision for the size and the composition of boards to maximize the market value of a firm. Also, this paper addresses a new question, a relationship between the quality of a CEO and the board structure. In my model, (1) the monitoring is to find precise information for the quality of a CEO, (2) the impact of advising by boards increases with the quality of a CEO (complementarity), and (3) there is "size (capital)-skill complementarity" on the role of a CEO. My model suggests that the size of boards increases with the size of a firm. The size of board is larger at the firm with less uncertainty if the uncertainty decreases the informational gain by boards. Finally, when the CEO is expected to be good, there is a "trade-off" between monitoring and advising benefit. Given the size of boards, the firms with more uncertainty and with talented CEOs allocate the more (less) directors to the advising (monitoring) committee. These predictions well explain the observed variations on boards.

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1   Introduction

There is a growing body of literatures which study the structure of boards. Hermalin and Weisbach (1998), Raheja (2005), and Adams and Ferreira (2007) provide theoretical models to explain the determinants of board structure. They commonly focus on the independence of boards (the proportion of outside directors on boards) by assuming that the insiders are well-known about firm-specific information, but enjoy more private benefit than outsiders. It naturally follows that insiders have an advantage in advising and outsiders in monitoring, so the benefit and cost of monitoring and advising are the main factors of board structures. Adams (2003), Lehn, Patro and Zhao (2004) and Linck, Netter and Yang (2006a) find empirical evidences that the size and the composition of boards depend on the characteristic of a firm. For instance, Adams (2003) shows that there is a fair amount of variation across firms in the amount of effort boards devote to their different roles, monitoring and advising. The large firms and the firms with more uncertainty devote more (less) effort to advising (monitoring). The boards of diversified firms devote relatively more (less) effort to monitoring (advising). Linck, Netter and Yang (2006a) show that the boards is larger and more independent at bigger firms. The firms with high growth opportunities and high stock return volatility have smaller and less independent boards.

In this paper, I, firstly, develop theoretical model to provide a rationale for the observed structure of boards. In my model, the firm (or shareholder) chooses the size and the composition of boards (inside directors VS outside directors or advising committee VS monitoring committee) to maximize the market value of a firm, which is differentiated with the previous "game theoretical" approach. Hermalin and Weisbach (1998) construct the bargaining model between CEO and boards who have different preference to explain the optimal independence of boards. Raheja (2005) provides "sequential game" among CEO, insider, and outsider (who have interest conflict) to predict the size and composition of boards. Similarly, Adams and Ferreira (2007) develop the applied model of strategic information transmission among CEO, insider, and outsider to argue that the CEO-friendly boards (or less independent boards) could be optimal under the certain circumstances. My model, however, does not take into account the strategic interaction among players. Only the firm (or shareholder) has strong power to make a decision, which reflects the current change in the environment around the corporate governance, for instance, the shareholder activism and the increasing market pressure for good governance. Then, in a different angle with the previous theories (Hermalin and Weisbach (1998), Raheja (2005), and Adams and Ferreira (2007)), I explain the observed

structure of boards, for instance, "What would be a channel through which the size of a firm affects boards?", "Why the more uncertainty makes monitoring less attractive?" Someone might argue that the firms with more uncertainty could extract more benefit to monitor the CEO effectively. It is also ambiguous whether these factors have effects on the benefit of monitoring or advising or both. To go further, we can ask a new question. If the CEO is expected to be good, what would happen to the structure of boards?

In my model, the quality of boards has a direct (advising CEO) and an indirect (monitoring CEO) mechanism to affect the earning of a firm. The monitoring role is defined by "finding precise information for the quality of an incumbent CEO and making turn-over decision". Also, the highly qualified monitoring committee increases the probability of finding precise information. The advising role is to guide managerial behavior. In terms of advising, I consider "complementality" between the CEO quality and the board quality in the production function, which implies that they interact each other positively.[1] Also, the impact of CEO's quality to the earning of a firm increases in the size of a firm under his control[2], which is originally introduced by Gabaix and Landier (2008).[3] The reason for incorporating this assumption into my model is that the role of boards is to monitor and to advise CEO, so if the quality of CEO matters more at bigger firms, the quality of boards could be more valuable there. Finally, the quality of boards (and each committee) increases in the number of directors.

My model provides the reason why the size of boards is larger at bigger firms. The directors monitor an incumbent CEO's action and then replace him with new CEO if the observed quality of an incumbent CEO is too low. Also, they give managerial advices to CEO. It naturally follows that the marginal impact of

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[1]Of course, it could be the case that the CEO can dominate decision-making or the decision of a CEO can be overridden by the board. Lee (2007b), however, finds that the board pay is 0.13% higher where CEO pay is 1% higher, which implies the complementarity between CEO and boards.

[2]We can interpret this assumption in the following manners: (1) The real power comes from the amount of resource which the CEO can allocate. (2) The "Size-Skill Complementarities" exists in the hierarchies of firm. Garicano and Rossi-Hansberg (2006) suggest that the ability of managers could be amplified by the amount of controllable resource in the hierarchies of firm because talented employees can share their ability (or knowledge) with the team under their control. In what follows, the more talented employees hold higher positions in the equilibrium.

[3]They propose a simple competitive assignment model in the CEO market to explain CEO compensation. They assume that CEOs have heterogeneous talent levels and are assigned to firms competitively. Also, the managerial impact of a CEO's talent increases with the value of the firm under his control. Under these assumptions, they suggest that the best CEO goes to the largest firms because the largest firm pays the most to the best CEO. Their empirical findings support these predictions. The CEO's pay increases with the size of a firm and the size of an average firm in the economy. Put simply, the most important determinant of the CEO compensation is the size of a firm, especially market capitalization.

boards to the earning of a firm is highly associated with the importance of CEO in the firm. Where the CEO's impact to the earning of a firm is stronger, the impact of directors is stronger, too. Since there assumed to be "size-quality(skill) complementarity" on the role of CEO, the CEO matters more at bigger firms. So does the director.

Secondly, the uncertainty of firms matters in a sense that it affects the marginal productivity of monitoring CEO. There is a possibility that the more uncertainty would make the monitoring role less attractive. As I mentioned before, the benefit of good boards is to find precise information for the quality of an incumbent CEO. Then, the directors are able to make a good "turn-over" decision to improve the market value of a firm. If the boards, even though they are experts, could not obtain precise information for the managerial action due to the uncertainty, the firms have less incentive to hire more directors. We can also interpret the first case in a following manner. The firms (or shareholders outside firms) already had prior information for the quality of a CEO. The reason why the firms hire directors is to obtain posterior information which is more precise than prior information. If there is no significant difference between prior and posterior information (if the posterior information is not much more precise than the prior information), the firms do not have to make a big boards. In a mean while, it could be the case that the uncertainty has positive effects on the marginal productivity of monitoring. Suppose that only the prior information for CEO becomes more noisy (or less precise) due to the uncertainty. Then, the firm (or shareholders outside the firm) would like to hire more directors to increase the probability of finding posterior information for CEO, because the posterior information (obtained by directors) becomes relatively more precise than the prior information. Put simply, the firm could enjoy more informational gain through boards. The first channel predicts that the size of board is smaller when the firms face more uncertainty. Conversely, the second channel shows that the size of board would increase in the uncertainty faced by firms.[4]

Thirdly, when the CEO is expected to be good, the firms face "trade-off" when they make decisions about the structure of boards. There is little incentive to find precise information for the quality of CEO by having more directors. The advising role, however, becomes more important owing to "complementarity" between CEO and boards. The good CEO and good directors could interact positivley. So, when the loss of monitoring benefit dominates the gain of advising role, the size of boards is smaller at the firms with more talented CEO and vice versa.

Given the size of boards, the firms with more uncertainty allocate the more

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[4]The empirical studies for the structure of boards support the first channel.

(less) directors to the advising committee or inside directors (monitoring committee or outside directors) if the boards, even though they are experts, could not obtain precise information for the managerial action due to the uncertainty. The uncertainty makes the benefit of monitoring small. The firms with talented CEOs allocate the more (less) directors to the advising (monitoring) committee. Weisbach and Hermalin (1988) find that the probability of hiring new independent diretors rises following poor firm performance. Hermalin and Weisbach (1998) theoretically argue that poor performance reduces the bargaing power of CEO in the selection process of directors, so that the CEO will be forced to accept more outside directors. My model could explain this in a different way. The quality of CEO is negatively associated with the marginal productivity of monitoring, but positively associated with the marginal productivity of advising. If the CEO is expected to be bad due to poor performance, the role of monitoring matters more, so that the firm will have more outside directors.

2 Related literature

As we discussed above, this paper is closely related with a field which studies the structure of boards. Maug (1997) argues that the firms with high information asymmetry would be more likely to incur the high cost of monitoring by independent directors because it is not easy to transfer firm-specific information to outsider directors. In this sense, it might not be optimal to have much independent directors. Hermalin and Weisbach (1998) construct the bargaining model between the CEO and the boards to fill vacancies on the boards. The CEO's bargaining power depends on his perceived ability and these nogotiations determine the independence of boards. For instance, the probability of inside directors being added to the boards increases when the firm performed well because the CEO's bargaining power becomes strong. Adams (2003) studies the variation of board composition by using data on 1542 board committees of 352 Fortune 500 companies in 1998. The boards of larger firms and firms facing more uncertainty devote relatively less effort to monitoring. The diversified firms, however, allocate relatively more effort to monitoring. Lehn, Patro and Zhao (2004) show that the size of board is positively associated with the firm size, but negatively related to growth opportunities by looking at 81 firms surviving from 1935-2000. Also, the bigger firms have more outside directors and the firms with more growth opportunities have less outside directors. Raheja (2005) argues that as the benefits of advising increase, boards will do more advising by having more inside directors. The reason is that the inside directors have good firm-specfic information, but outside directors are usu-

ally lack of knowledge. Adams and Ferreira (2007) develop a model by applying strategic information transmission game between the CEO and the boards. The CEO faces a trade-off when he discloses information to the directors. To reveal information makes the quality of advise by boards better, but, simultaneously, the level of monitoring tougher. Under certain circumstances, the CEO do not want to share information with directors, so that CEO-friendly boards (or less independent boards) could be optimal. Linck, Netter and Yang (2006a) provide empirical evidence for the structure of boards. Using board profiles for 7000 firms from 1990 to 2004, they show that the board size increases in the size of a firm, but decreases with high growth opportunities, high R&D expenditures, and high stock return volatility. Also, the firms with high managerial ownership have smaller boards.

3   Model

Based on Hermalin and Weisbach (1998), Gabaix and Landier (2008) and Lee (2007b), I set up the timing of this model for the following. At the first stage, the firm only has the prior distribution of the quality of an incumbent CEO, q^c, which has mean μ and variance $\frac{1}{\tau}$. In the second stage, the firm would be likely to have nothing for the quality of an incumbent CEO. In this case, the firm retains a current CEO. At the same time, the firm also has the possibility that it can obtain precise information for the quality of incumbent CEO denoted by g. The distribution of g given the CEO's ture quality, q^c, has mean q^c and variance $\frac{1}{t}$ which is assumed to be less than $\frac{1}{\tau}$. The probability of finding precise information depends on the quality of the monitoring committee, $q_n(n_m)$, where n_m is the number of directors allocated to the monitoring committee.[5] Simply, $\frac{\partial q_m(n_m)}{\partial n_m} > 0$. In contrast, the firm can only find nothing for the quality of an incumbent CEO with probability $1 - q_m(n_m)$. Here, I assume that total number of directors on boards is N, so that

$$n_m + n_a = N$$

where n_a is the number of directors allocated to the advising committee. Finally, the firm decides whether to fire the incumbent CEO or not based on informations g or the prior expectation of the quality of an incumbent CEO, and then the market value of a firm is realized. If the incumbent CEO is fired, the market value of a firm depends on the quality of a replaced CEO. The quality of a potentially replaced

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[5]In this paper, I assume that the quality of each committee depends on the number of board member assigned to each commitee. However, we can also assume that the quality depends on the effort devoted to each committee followed by Adams (2003).

CEO is randomly distributed with mean $\mu_R < \mu$, [6] and variance $\frac{1}{\tau_r}$. For the sake of exposition, all random variables are assumed to follow normal distribution.

As I assumed before, the firm knows that the CEO's ability is drawn from a normal distribution with mean μ and variance τ at the first stage, which is the prior distribution of the CEO's quality. If the firm obtains precise information for the quality of an incumbent CEO, denoted by g, which is assumed to be distributed with mean q^c (true quality of an incumbent CEO) and variance $\frac{1}{t}$, then the posterior expectation of CEO's quality is

$$E[q^c \mid g] = \frac{\tau\mu + gt}{\tau + t}$$

The precision is $\tau + t$. If the firm finds precise information, the incumbent CEO is fired when

$$E[q^c \mid g] = \frac{\tau\mu + gt}{\tau + t} < \mu_R$$

Then, we can find cutoff level G by

$$g < -\frac{\tau\mu}{t} + \frac{(\tau + t)\mu_R}{t} \equiv G$$

In other words, if $g < G$, then the incumbent CEO is fired. When the firm finds nothing, the incumbent CEO should be retained because the firm decides whether to fire CEO or not based on the prior expectation, $\mu > \mu_R$.

3.1 The Optimal Size of Boards

Suppose that the firm only chooses the size of boards to maximize the market value of a firm, and the probability of finding precise information for the quality of an incumbent CEO is equal to the quality of boards, $Q_B(N)$, where N is the size of boards (the number of directors). Simply, $\frac{\partial Q_B(N)}{\partial N} > 0$. In contrast, the firm is only able to find nothing with probability $1 - Q_B(N)$. All other things are same as above.

3.1.1 The market value of firms

CEO quality and Board quality Now, I will derive the market value of a firm which depends on the quality of CEO and boards. I assume the symmetric complementarities (Becker (1981) and Becker (1993)) between CEO quality and

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[6]This assumption guarantees that an incumbent CEO will not be fired when the firm does not get any information.

board quality.[7] Based on Murphy and Zabojnik (2004) and Gabaix and Landier (2008), the market value of a firm is given by

$$M[q^k, Q_B] = \underbrace{S^\sigma \left(E[q^k]Q_B(N) \right)}_{\text{Earning}} - \underbrace{W_{CEO} - NW_{board}}_{\text{Cost}}, \ k = u \text{ or } r \qquad (1)$$

where S denotes the size of a firm,[8] σ "return to scale" parameter which is $0 < \sigma < 1$ and q^k the quality of CEO. If an incumbent CEO is retained, the market value of a firm depends on the quality of an incumbent CEO, q^u. Otherwise, the market value of a firm depends on the quality of a replaced CEO, q^r. $Q_B(N)$ is the quality of boards which is $\frac{\partial Q_B(N)}{\partial N} > 0$. The cost is composed of two parts, the CEO pay, W_{CEO}, and the pay for boards, W_{board}. N is the size of boards. More specifically, the market value of a firm is expressed by

$$
\begin{aligned}
M[q^k, Q_B] \ &= \ S^\sigma \left(E[q^k]Q_B(N) \right) - W_{CEO} - NW_{board} \\[2mm]
&= \ \underbrace{S^\sigma Q_B(N) \left\{ \int\limits_{G}^{\infty} \left(\frac{\tau\mu + gt}{\tau + t} \right) f_g(g)dg + \int\limits_{-\infty}^{G} \mu_R f_g(g)dg \right\} Q_B(N)}_{\text{expected earning from precise information}} \\[2mm]
&\quad + \ \underbrace{S^\sigma (1 - Q_B(N)))\mu Q_B(N)}_{\text{expected earning from no information}} - W_{CEO} - NW_{board}, \ k = u \text{ or } r
\end{aligned}
$$

The firm solves N to maximize the market value of a firm

$$\max_N \ M[q^k, Q_B(N)] = \Omega Q_B^2(N) + \Psi Q_B(N) - W_{CEO} - NW_{board} \qquad (2)$$

where

$$\Omega = S^\sigma \left[\mu H(-\sqrt{T}(G - \mu)) + \frac{\sqrt{T}}{\tau} h(\sqrt{T}(G - \mu)) + \mu_R H(\sqrt{T}(G - \mu)) - \mu \right]$$

and

$$\Psi = S^\sigma \mu$$

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[7] Alternatively, we can consider the production function which is asymmetrically sensitive to the quality of CEO and of boards, for instance,

$$S^\sigma (E[q^k])^\theta (\overline{q_n})^{1-\theta}$$

This is originally proposed by Kremer and Maskin (1996). This, however, does not affect the qualitative prediction of model in this paper.

[8] This set-up reflects the "Size-Skill Complementarity". See Gabaix and Landier (2008) for more details.

$H(\cdot)$ follows a standard normal distribution,[9] and T is given by $\frac{1}{\frac{1}{\tau}+\frac{1}{t}}$. For the sake of exposition, I assume that

$$Q_B(N) = \alpha(N)^\gamma, \ 2\gamma < 1 \tag{3}$$

Also, I assume that "return to scale" parameter, σ, is equal to 1. Plugging equation (3) into (2), we can get the market value of a firm by

$$M[q^k, Q_B(N)] = \Omega\alpha(N)^{2\gamma} + \Psi\alpha(N)^\gamma - W_{CEO} - NW_{board}, \ k = u \text{ or } r \tag{4}$$

Taking derivatives of equation (4) with respect to N, we can get the first order condition by

$$2\alpha^2\Omega 2\gamma N^{2\gamma-1} + \alpha\Psi N^{\gamma-1} = W_{board} \tag{5}$$

where the left-hand side of above equation represents the marginal impact of boards to the earning of a firm.

3.1.2 Comparative Statics

For the sake of exposition, I, hereafter, assume that the expectation for the quality of a replaced CEO, μ_R, is equal to zero.

Proposition 1 *The equilibrium size of boards would increase in the size of a firm.*

Proof. Taking derivatives of equation (5) with respect to the size of a firm, S, we can get

$$2\alpha^2\frac{\partial\Omega}{\partial S}2\gamma N^{2\gamma-1} + 2\alpha^2\Omega 2\gamma(2\gamma-1)\left(\frac{\partial N}{\partial S}\right)^{2\gamma-2}$$

$$+\alpha\frac{\partial\Psi}{\partial S}N^{\gamma-1} + \alpha\Psi(\gamma-1)\left(\frac{\partial N}{\partial S}\right)^{\gamma-2} = 0$$

Since

$$\frac{\partial\Omega}{\partial S} = \left\{\mu H(-\sqrt{T}(G-\mu)) + \frac{\sqrt{T}}{\tau}h(\sqrt{T}(G-\mu))\right\} > 0$$

and

$$\frac{\partial\Psi}{\partial S} = \mu > 0,$$

then $\frac{\partial N}{\partial S} > 0$, which implies that the size of boards is larger at bigger firms ∎

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[9]Note that $H(-\sqrt{T}(G-\mu))$ represents the probability of incumbent CEO's retaining when precise information are obtained. See Hermalin (2005), page 2357.

The boards monitors and advises CEO to affect the market value of a firm. So, the marginal impact of boards to the earning of a firm is highly associated with the importance of CEO at the firm. Since there assumed to be "size-quality(skill) complementarity" on the role of CEO, the CEO matters more at bigger firms. So does the director.

Proposition 2 *The equilibrium size of boards would decrease in the prior expectation for the quality of an incumbent CEO if $\frac{\partial \Omega}{\partial \mu}$ is small enough.*

Proof. Taking derivatives of equation (5) with respect to the prior expectation for the quality of an incumbent CEO, μ, we can get

$$2\alpha^2 \frac{\partial \Omega}{\partial \mu} 2\gamma N^{2\gamma-1} + 2\alpha^2 \Omega 2\gamma(2\gamma - 1)\left(\frac{\partial N}{\partial \mu}\right)^{2\gamma-2}$$

$$+\alpha \frac{\partial \Psi}{\partial \mu} N^{\gamma-1} + \alpha \Psi(\gamma - 1)\left(\frac{\partial N}{\partial \mu}\right)^{\gamma-2} = 0$$

In Hermalin (2005), we can find

$$\frac{\partial\left(\mu H(-\sqrt{T}(G-\mu)) + \frac{\sqrt{T}}{\tau}h(\sqrt{T}(G-\mu))\right)}{\partial \mu}$$

$$= H(-\sqrt{T}(G-\mu)) + \mu\sqrt{T}h(-\sqrt{T}(G-\mu)) + \frac{T\sqrt{T}(G-\mu)}{\tau}h(\sqrt{T}(G-\mu))$$

$$= H(-\sqrt{T}(G-\mu))$$

Then, it can be easily seen that

$$\frac{\partial \Omega(\mu, S)}{\partial \mu} = S\left[H(-\sqrt{T}(G-\mu)) - 1\right]$$

Since
$$\frac{\partial \Omega(\mu, S)}{\partial \mu} = S\left[H(-\sqrt{T}(G-\mu)) - 1\right] < 0 \text{ and } \frac{\partial \Psi(\mu, S)}{\partial \mu} > 0,$$

then if $2\alpha^2 \frac{\partial \Omega}{\partial \mu} 2\gamma N^{2\gamma-1} + \alpha \frac{\partial \Psi}{\partial \mu} N^{\gamma-1} < 0$, $\frac{\partial N}{\partial \mu} < 0$ ∎

When the CEO is expected to be good, there is a "trade-off" between monitoring and advising benefit. Since, from the viewpoint of firms, there is little incentive to find precise information for the quality of CEO, the monitoring role of boards

becomes less important. The advising role, however, becomes more important owing to "complementarity" between CEO and boards. The impact of good CEO to the earning of a firm could be amplified by hiring good directors. So, when the loss of monitoring benefit dominates the gain of advising role, the size of boards is smaller at the firms with more talented CEO.

Proposition 3 *The equilibrium size of boards would decrease in the precision of prior information, τ and would increase in the precision of posterior information, t*

Proof. omitted ∎

Now, we discuss how to incorporate the uncertainty of a firm into this model. In general, the uncertainty (faced by firms) is closely related with the precision of information. One way is to assume that the uncertainty only affects the precision of prior information for the quality of an incumbent CEO, τ, not the precision of posterior information, t. Suppose that the prior information for CEO becomes more noisy (or less precise) due to the uncertainty. Then, the firm (or shareholders outside the firm) would like to hire more directors to increase the probability of finding posterior information for CEO, because the posterior information (only obtained by directors) becomes relatively more precise than the prior information. Put simply, the firm could enjoy more informational gain through boards, so the size of boards is larger at the firms with more uncertainty.

However, it could be the case that the posterior information only becomes more noisy (or less precise) due to the uncertainty. The uncertainty would make the directors, even though they are experts, hard to find precise information for the managerial action. Then, the firm (which already had the prior information) is willing to hire less directors because the posterior information is not much precise than the prior information. The size of boards is smaller at the firms with more uncertainty.

3.2   Optimal Allocation of Boards

Now, I will derive the market value of a firm which depends on the quality of CEO, monitoring committee (or outside director) and advising committee (or inside director). The market value of a firm is given by

$$M[q^k, q_a(n_a), q_m(n_m))] = S^\sigma \left(E[q^k] q_a(n_a) \right) - W_{CEO} - (n_m + n_a) W_{board}, \ k = u \text{ or } r$$

where S denotes the size of a firm, σ "return to scale" parameter which is $0 < \sigma < 1$, and q^k the quality of CEO. Also, $q_a(n_a)$ is the quality of advising committee(or

inside director) which is assumed to be $\frac{\partial q_a(n_a)}{\partial n_a} > 0$ and $q_m(n_m)$ is the quality of monitoring committee (or outside director) which is assumed to be $\frac{\partial q_m(n_m)}{\partial n_m} > 0$. Finally, n_a and n_m represent the number of advising committee (or inside director) and monitoring committee (or outside director), respectively. More specifically, the market value of a firm is expressed by

$$
M[q^k, q_a(n_a), q_m(n_m))] \;=\; S^\sigma \left(E[q^k] q_a(n_a) \right) - W_{CEO} - (n_m + n_a) W_{board}
$$

$$
\;=\; S^\sigma q_m(n_m) \underbrace{ \left\{ \int\limits_{G}^{\infty} \left(\frac{\tau\mu + gt}{\tau + t} \right) f_g(g) dg + \int\limits_{-\infty}^{G} \mu_R f_g(g) dg \right\} q_a(n_a) }_{\text{expected earning from precise information}}
$$

$$
\;+\; \underbrace{ S^\sigma (1 - q_m(n_m)) \mu q_a(n_a) }_{\text{expected earning from no information}} \;-\; W_{CEO} - (n_m + n_a) W_{board}
$$

$$
k \;=\; u \text{ or } r
$$

Then, the firm solves n_m, n_a to maximize the market value of a firm

$$
\max_{n_m, n_a} \; M[q^k, q_a(n_a), q_m(n_m))] \tag{6}
$$

$$
\;=\; \Omega q_m(n_m) q_a(n_a) + \Psi q_a(n_a) - W_{CEO} - (n_m + n_a) W_{BOARD}, \; k = u \text{ or } r
$$

$$
s.t \; n_m + n_a = N
$$

where Ω, Ψ, T, A and $H(\cdot)$ are same as above. Then, the marginal impact of monitoring committee (outside directors) and advising committee (inside directors) to the earning of a firm is defined by

$$
\Omega q_a(n_a) \frac{\partial q_m(n_m)}{\partial n_m} + \Omega q_m(n_m) \frac{\partial q_a(n_a)}{\partial n_m} + \Psi \frac{\partial q_a(n_a)}{\partial n_m}
$$

and

$$
\Omega q_m(n_m) \frac{\partial q_a(n_a)}{\partial n_a} + \Omega q_a(n_a) \frac{\partial q_m(n_m)}{\partial n_a} + \Psi \frac{\partial q_a(n_a)}{\partial n_a}
$$

For the sake of exposition, I assume that (1) $q_m(n_m) = \alpha(n_m)^\gamma$, $q_a(n_a) = \beta(n_a)^\gamma$, and $2\gamma < 1$, (2) W_{CEO} and W_{board} are given, and (3) "return to scale" parameter, σ, is equal to 1. Then, we can rewrite the equation (6) by

$$
\Omega\alpha\beta(n_m N - n_m^2)^\gamma + \Psi\beta(N - n_m)^\gamma - W_{CEO} - NW_{board}, \; k = u \text{ or } r \tag{7}
$$

Taking derivatives of equation (7) with respect to n_m, we can get the first order condition by

$$\Omega\alpha\beta\gamma(n_m)^{\gamma-1}(N-2n_m)-\Psi\gamma\beta=0 \tag{8}$$

Proposition 4 *Given the size of boards, the equilibrium number of monitoring committee (or outside directors) would decrease with the prior expectation of incumbent CEO's quality.*

Proof. Taking derivatives of equation (8) with respect to μ, we can get

$$\frac{\partial\Omega}{\partial\mu}\alpha\beta\gamma(n_m)^{\gamma-1}(N-2n_m)$$

$$+\Omega\alpha\beta\gamma\left(\frac{\partial n_m}{\partial\mu}\right)(n_m)^{\gamma-2}\{(\gamma-1)(N-2n_m)-2n_m\}$$

$$=\frac{\partial\Psi}{\partial\mu}\beta$$

Since $\frac{\partial\Omega}{\partial\mu}<0$ and $\frac{\partial\Psi}{\partial\mu}>0$, then $\frac{\partial n_m}{\partial\mu}<0$ ■

Simply, we can interpret the marginal impact of monitoring committee (or outside directors) to the earning of a firm as the expected earning from precise information minus the expected earning from nothing. When the CEO is expected to be good, the monitoring role becomes less important, but the advising role becomes more important due to the complementarity between CEO and boards. So, the equilibrium number of monitoring committee (or outside directors) would decrease with the prior expectation of incumbent CEO's quality. The boards is less independent at the firms with more talented CEO.

Proposition 5 *The equilibrium number of monitoring committee (or outside directors) would decrease with the size of a firm if $\frac{\left(\frac{\partial\Psi}{\partial S}\right)}{\left(\frac{\partial\Omega}{\partial S}\right)}>\alpha\gamma(n_m)^{\gamma-1}(N-2n_m)$. Otherwise, the equilibrium number of monitoring committee (outside directors) would increase with the size of a firm*

Proof. Taking derivatives of equation (8) with respect to S, we can get

$$\frac{\partial\Omega}{\partial S}\alpha\beta\gamma(n_m)^{\gamma-1}(N-2n_m)$$

$$+\Omega\alpha\beta\gamma\left(\frac{\partial n_m}{\partial S}\right)(n_m)^{\gamma-2}\{(\gamma-1)(N-2n_m)-2n_m\}=\frac{\partial\Psi}{\partial S}\beta$$

Rearranging above equation,

$$\left(\frac{\partial n_m}{\partial S}\right) = \frac{\frac{\partial \Psi}{\partial S}\beta - \frac{\partial \Omega}{\partial S}\alpha\beta\gamma(n_m)^{\gamma-1}(N-2n_m)}{\Omega\alpha\beta\gamma(n_m)^{\gamma-2}\left\{(\gamma-1)(N-2n_m)-2n_m\right\}}$$

So, if

$$\frac{\left(\frac{\partial \Psi}{\partial S}\right)}{\left(\frac{\partial \Omega}{\partial S}\right)} > \alpha\gamma(n_m)^{\gamma-1}(N-2n_m),$$

then the number of monitoring committee (outside directors) would decrease in the size of a firm ∎

The size of a firm affects both the impact of monitoring (or outside directors) and advising (or inside directors) to the earning of a firm. My model assumes that the impact of CEO to the earning of a firm increases with the size of a firm under his control. CEO matters more at bigger firms, so that monitoring and advising CEO matter more, too. The allocation of board members (or the independence of boards) would be determined by the relative effect of the firm size on the impact of monitoring and advising to the earning of a firm.

Proposition 6 *The equilibrium number of monitoring committee (outside directors) would decrease in the precision of prior information, τ and would increase in the precision of posterior information, t*

Proof. Omitted ∎

Suppose that the prior information for CEO becomes less precise (low τ) due to the uncertainty. Then, the firm (or shareholders outside the firm) would like to allocate more directors to monitoring committee (or make the boards more independent) to increase the probability of finding posterior information for CEO, because the posterior information obtained by monitoring committee (or outside directors) becomes relatively more precise than the prior information. Put simply, the firm could enjoy more monitoring benefit to allocate more directors to monitoring committee (or increase the proportion of outside directors on boards). However, it could be the case that the posterior information only becomes less precise (low t) due to the uncertainty. Then, the firm (which already had the prior information) has an incentive to allocate less directors to monitoring committee (or decrease the proportion of outside directors on boards).

4   Conclusion

This paper provides a theoretical background for (1) the size of boards and (2) the allocation of board members to each committee, monitoring committee and

advising committee. This model can be applied to explain the indepedence of boards, too. My model shows the channels through which the size of a firm, the uncertainty faced by a firm, and the expectation for the quality of a CEO affect the structure of boards by defining the marginal productivity of monitoring and of advising. Also, we discuss how to incorporate the uncertainty and the asymmetric information between inside and outside the firm into the model. Overall, my predictions well explain the observed structure of boards. The size of boards increases with the size of a firm. The size of board is larger where the firms face less uncertainty. Finally, when the CEO is expected to be good, there is a "trade-off" between the benefit of monitoring and advising. If the gain in the benefit of advising dominates the loss in the benefit of monitoring, the size of boards is bigger at the firms with talented CEO. Given the size of boards, the firms with talented CEOs and with more uncertainty allocate the more (less) directors to the advising (monitoring) committee. We can also expect that the independence of boards is negatively associated with the quality of CEO and the uncertainty faced by firms.

References

Adams, Renee B. 2003. "What do boards do? Evidence from Board Committee and Director Compensation Data." *EFA 2005 Moscow Meetings Paper* .

Adams, Renee B. and Daniel Ferreira. 2007. "A Theory of Friendly Board." *Journal of Finance, Forthcomming* 62:217–250.

Becker, Gary. 1981. *Treatise on the Family.* Cambridge: Harvard University Press.

Becker, Gary. 1993. *Human Capital.* Chicago: University of Chicago Press.

Gabaix, Xavier and Augustin Landier. 2008. "Why Has CEO Pay Increased So Much?" *Quarterly Journal of Economics* 123:49–100.

Garicano, Luis and Esteban Rossi-Hansberg. 2006. "Organization and Inequality in a Knowledge Economy." *Quarterly Journal of Economics* pp. 1383–1435.

Hermalin, Benjamin E. 2005. "Trends in Corporate Governance." *Journal of Finance* pp. 2351–2384.

Hermalin, Benjamin and M. Weisbach. 1998. "Endogenously Chosen Boards of Directors and Their Monitoring of Management." *American Economic Review* 88:96–118.

Kremer, Michael and Eric Maskin. 1996. "Wage Inequality and Segregation by Skill." *NBER Working Paper No. 5718* .

Lee, Changmin. 2007b. "The Market for Corporate Directors." *CAEPR Working Paper* available at SSRN: http://ssrn.com/abstract=988119(2007-006).

Lehn, Kenneth, Sukesh Patro and Mengxin Zhao. 2004. "Determinant of the size and structure of corporate boards:1935-2000." *Working Paper* .

Linck, James S., Jeffry M. Netter and Tina Yang. 2006a. "The Determinant of Board Structure." *Working paper Available at SSRN: http://ssrn.com/abstract=729935* .

Maug, Ernst. 1997. "Boards of directors and capital structure: Alternative forms of corporate restructuring." *Journal of Corporate Finance* 3:113–139.

Murphy, Kevin and Jan Zabojnik. 2004. "CEO Pay and Appointment: A Market-Based Explanation for Recent Trends." *American Economic Review Papers and Proceedings* 94(2):192–196.

Raheja, Charu G. 2005. "Determinants of board size and composition." *Journal of Financial and Quantitative Analysis* 40:283–306.

Weisbach, Michale and B. Hermalin. 1988. "The Determinant of Board Composition." *RAND Journal of Economics* 19:589–606.