Unbalanced Matching in a Director Market

Changmin Lee*
Indiana University - Bloomington

12th October 2008

Abstract

The CEO's of relatively big firms in U.S. tend to serve as outside directors on firms smaller than their own. Also, the best director on boards of American large firms is the CEO's of bigger firm. To explain these, I construct an intertemporal searching model with heterogeneous directors and firms in a director market. Both sides only care about quality, not price. My calibration shows that the best candidate for outside directors would be willing to accept an offer from the lower 40% firm (e.g., 300th best of 500 firms) under the uniform distribution of a firm, and from the lower 46% (e.g., 280th best of 500 firms) under the extreme value distribution.

JEL Classification: G34, G38, J41, J44, J64

Keywords: Corporate Governance, Board of Directors, Matching Model, Job Search

*Author: Changmin Lee, Department of Economics, Wylie Hall, Indiana University, 47405, IN, e-mail: cl3@indiana.edu. I would like to thank Eric Rasmusen for many helpful discussions. I also thank Michael Alexeev, Richard Shockley and Konstantin Tyurin.

1 Introduction

Gabaix and Landier (2008) propose a simple competitive assignment model to explain CEO compensation. They assume that CEOs have heterogeneous talent levels and are assigned to firms competitively. The managerial impact of a CEO's talent increases with the value of the firm under his control. Under these assumptions, they find that the best CEO (a CEO with the most talent) has the highest marginal productivity in the largest firm, and the largest firm will pay the highest wage. Simply stated, the best CEO goes to the largest firms and a CEO's pay increases with the size of a firm and of an average firm in the economy. Their empirical findings support these predictions.

In this paper, I explore the matching pattern in the market for corporate directors. To that purpose, I analyze board profile of 250 U.S. firms among Fortune 500 firms in 2005. Then, I rank all ongoing and retired CEOs who are working as outside directors on boards based on the size of a firm. Suppose that A, an outside director on boards of Wal-Mart is an ex-CEO of GM. The quality of A is measured by the size (market capitalization) of GM. If GM is the largest firm in my data set, A director is the best candidate in a director market. Similarly, the quality of a firm (say, Wal-Mart) is measured by the size (market capitalization) of a firm.

There are some stylized facts in the market for corporate directors. First, the average quality of boards[1] is not significantly different with the size of a firm. For instance, the average director of GE is from a similar-sized firm as GE. Second, the quality of boards is higher at those firms with larger market capitalization, which implies that (ongoing or retired) CEOs of big firms also work at big firms as outside directors. (See Lee (2007b)) Third, the quality of directors on the same boards is very dispersed. For instance, seven outside directors on boards of the upper 10% ranked firms (very big firms in my sample) are ongoing or retired CEO's, and their quantile is 8%, 19%, 22%, 31%, 33%, 40% and 43%. Similarly, the quantile of seven outside directors on boards of 60% ranked firm is 58%, 65%, 67%, 69%, 76%, and 82%. Finally, the talented directors (CEOs of relatively big firms) serve as outside directors on firms smaller than their original firms. My data shows that the quality of upper 0%~25% ranked directors is significantly higher than the quality of matched firms. Also, the best director of each firm tends to be the CEO's of more large firms. For instance, the best director on boards of Target is from a bigger firm than Target. (See *Table 1-A* and *1-B*)

[1] The average quality of boards is calculated as follows: suppose that there are three outside directors (A, B, and C) on boards of GE. Then,

$$\text{the average quality of GE boards}$$
$$= \frac{\text{the size of A's own firm+B's own firm+C's own firm}}{3}$$

I test the equality of matched pairs of observations (the average quality of boards and the size of a firm) by using "signtest" and "signrank" command in Stata.

Table 1 Descriptive statistics for the quality of directors

- *The quality of directors on 266 firms' boards among Fortune 500 U.S firms in 2005*
- *Proxy for the quality of boards and firms: the market capitalization*
- *All values are in $million*

	The quality of directors	*The quality of firms*
Mean	*82,399*	*82,649*
Standard deviation	*178,435*	*178,054*
Max	*1421862*	*1626213*
Min	*36.04*	*936.84*
25%	*6908.75*	*15208.44*
50%	*24701.89*	*28804.5*
75%	*63509*	*69040.79*

- *The average quality of boards is the averaged quality of directors on the same boards*
- *The "signrank" and "signtest" show that the average quality of boards is neither significantly higher nor lower than the quality of firms.*

	The average quality of boards	*The quality of firms*
Mean	*84,740*	*106,444*
Standard deviation	*122,331*	*214,810*
Max	*786,867*	*1626,213*
Min	*1006.19*	*3659*
25%	*21,721*	*19,234*
50%	*41,550*	*35,397*
75%	*105,962*	*84,637*

The main goals of this paper are (1) to explain "why do the high-ability CEO's often work at bad (not too bad) firms as outside directors?" and (2) to make a "calibratable model" which could produce quantitative predictions for the matching in the market for corporate directors. To this purpose, I construct an intertemporal searching and matching model ("take it or leave it offer") in a frictional directorship market under four main assumptions: (1) the candidates for outside directors want to maximize their reputation value which depends on the size of a firm;[2] (2) there is no wage competition or wage bargaining in a director market. It naturally follows that the firms simply prefer highly qualified directors; (3) the outside director positions are randomly opened

[2] Hambrick and Johnson (2000) said "The majority of outside directors are fully motivated to act conscientiously and vigorously by forces other than a financial stake in the firm: their sense of professionalism, concern for their reputations and stature, and the threat of lawsuit." (Colley and Stettinius (2003), page 61)

over the time, and (4) the potential candidates for outside directors and firms have heterogeneous quality levels and both sides only care about quality, not price. Also, a candidate considers his quality ranking in order to compare the value of accepting a favorite offer at present to the value of waiting for a better offer in the future.

Under a certain assumption, there is the possibility that highly qualified candidates are matched with bad (not too bad) firms. The best candidate would be willing to accept an offer from the lower 40% firm (e.g., 300th best of 500 firms) under the uniform distribution of a firm, and from the lower 46% (e.g., 280th best of 500 firms) under Gabaix and Landier (2008)'s extreme value distribution. Also, the 25th ranked candidate, for instance, would be willing to accept an offer from the lower 25% firm under the uniform distribution, and from the lower 30% under the extreme value distribution. Put all together, the high-ability directors, the 1st˜25th ranked ones, would be willing to accept an offer from lower 25%˜40% (30%˜46%) firms. This calibration approximately explains the matching pattern in the sample of high-ability directors. *Table 2* shows the case of unbalanced matching between high-ability directors (CEO's of relatively big firms) and bad firms (relatively small firms) in my sample. For instance, the 5th ranked director is matched with the lower 47% firm, the 32th ranked one with the lower 26%, and the 43th one with the lower 20% firm. Overall, the highly qualified candidates (the 1st˜25th ranked directors, upper 0%˜5% in my sample) are sometimes matched with lower 25%˜49% ranked firms, and, more broadly, the 1st˜50th ranked directors (upper 0%˜10% in my sample) with lower 20%˜49% firms.

Table 2 the case of unbalanced matching between high-ability directors (CEO's of relatively big firms) and bad firms (relatively small firms) in my sample

- *All directors here are upper 0%~10% ranked directors in my sample*
- *The quantile of matched firms is the lower quantile value in my sample*

The rank of directors	*The quantile of matched firms*
5	*lower 0.47*
6	*0.25*
21	*0.49*
32	*0.26*
33	*0.28*
34	*0.27*
37	*0.33*
38	*0.36*
39	*0.43*
43	*0.20*
47	*0.34*

There might be three alternative ways to explain these observed facts. First, we can argue that negative matching would take place in the market for corporate directors. The potential directors might want to work at low-valued firms because they like "importance" or "influence" in the firms. Lee (2007b), however, finds that the matching pattern is positive in a sense that the average, highest, and lowest quality of directors on each boards are higher where the firms are larger. Second, it might be that there simply are more good candidates for directors than there are good firms. Then, many bad firms will have good directors. However, Brickley, Linck and Coles (1999) and Lee (2007c) show that the ex-CEO's of big firms tend to have more outside directorships, which implies that good candidates are scarce in a director market. Suppose there are 30 big companies, and each of them has one CEO and one retired CEO who hold 2 outside directorships. That makes a supply of 120 directors. If each firm would like to have 6 outside directors, that creates a demand of 240 directors. Third, the quality measure might be noisy or incorrect. In this paper, I assume that the CEO's who are running or have run big firms are "high-ability" ones, but, for instance, the accounting performance (ROA and industry adjusted ROA) and the stock return during CEO tenure are two possible alternatives. Lee (2007c) analyzes directorships held by 250 CEOs who retired during 1998-2002 in the two years after retirement. He finds that the firm size (total assets) in which CEOs worked before retirement is directly related to the number of outside directorships,[3] but the accounting performance (ROA and industry adjusted ROA) and the stock return during tenure do not have a significant effect on it. We can interpret that the firms prefer retired CEOs from large firms as outside board members or the size of a firm matters more as an indicator of quality in the market for corporate directors.

The rest of the paper is organized as follows. Section 2 provides a brief review of the related literature. In section 3, I develop an intertemporal searching and matching model. Section 4 shows the numerical analysis. I summarize concluding remarks in Section 5.

2 Related literature

2.1 The matching/searching and matching

Gale and Shapley (1962) study the classic matching problem in the marriage market. They assume that each part (man and woman) ranks counterparts based on idiosyncratic preferences and show that there always exists a stable assignment equilibrium. In contrast, this paper assumes that both parties have the same preference order. Both parties only care about the quality of counterparts. They simply prefer high quality to low quality. McCall (1970) develops the intertemporal searching and matching model in the labor market. He sets up the model of a representative agent who lives forever and uses a sequential search technique to explain the determination of reservation wage. I

[3] Brickley, Linck and Coles (1999) also show that the firm size in which CEOs worked before retirement well explains the number of outside directorship held by CEOs 2 years after retirement.

also analyze the determination of reservation quality level based on the intertemporal searching and matching framework, but both parties in the matching market are heterogeneous and well-ranked in terms of quality. Gabaix and Landier (2008) propose a simple competitive assignment model to explain CEO compensation. They assume that CEOs have heterogeneous talent levels and are assigned to firms competitively. The managerial impact of a CEO's talent increases with the value of the firm under his control. Under these assumptions, they find that the best CEO goes to the largest firm and a CEO's pay increases in the size of firm and of an average firm in the economy. Their empirical finding supports these predictions. In this paper, I study the matching problem in a frictional director market. The vacancies for the outside directors are assumed to be randomly opened over time.

3 Model

I develop an intertemporal searching and matching model in which potential candidates for outside directorships live forever and are risk neutral. Based on Gale and Shapley (1962), McCall (1970) and Gabaix and Landier (2008), I construct the basic setting. There are m number of potential candidates for outside directorships with a well-ordered quality level, q_{new}^{k}. k denotes the ranking of his quality level in the pool of potential candidates. There is no possibility that $q_{new}^{i} = q_{new}^{j}$ when $i \neq j$. I assume that the number of potential candidates, m, is time-invariant on the steady-state, which means that the inflow into the potential candidates pool is equal to the outflow from the potential candidates pool. I also assume that the quality ranking of each potential candidate is time-invariant on the steady state.

At each time, the n number of firms each creates one vacancy for an outside director position. The n number of vacancies for an outside director position are randomly created in that the quality of firms, q_f, [4] is randomly drawn from a non-decreasing and continuous distribution $F(q_f)$ on $[0, \overline{q_f}]$. Also, $F(0) = 0$ and $F(\overline{q_f}) = 1$. The distribution for the quality of firms is well dispersed so that there is no possibility for $q_f^i = q_f^j$ when $i \neq j$. i and j denotes the quality ranking of firms. All firms have a strictly ordered preference over the quality of a candidate, say, $q_{new}^1 \succ q_{new}^2 \succ q_{new}^3$.... Also, all candidates have a strictly ordered preference over the quality of firms, say, $q_f^1 \succ q_f^2 \succ q_f^3$...

The stage is set for the following. At first, all firms make an offer to a favorite one (the first-ranked candidate). The first-ranked candidate rejects all but his favorite one and decides whether to accept this favorite offer or also reject this to allow for the possibility that a better offer may come along later. In the second stage, those firms who are rejected give offers to their second choice. The second-ranked candidate iterates this process. If $m \leq n$ all potential candidates will eventually have received an offer and made decisions. If $m > n$ only the first n number of potential candidates will have received a

[4] We can interpret the quality of a firm in three different ways: (1) the ranking of a firm in the economy, (2) the quality of a CEO, and (3) the size of a firm. The functional form of distribution $F(q_f)$, depends on interpretations.

certain offer and made a decision.[5] Below, I will focus on the steady state equilibrium in the latter case.[6]

3.1 The value function

If a candidate with the quality ranking k has a favorite offer in the first n number of job opening denoted by $q_{k,f}$, then he will make a decision whether to accept or wait for a better offer in the future. Neither quitting nor firing is allowed. At each period, a candidate chooses a strategy to maximize $E\sum_{t=0}^{\infty}(\frac{1}{1+r})^t y_t$, where y_t is the reputation value from working as outside director depending on $q_{k,f}$. The Bellman equation for the candidate's problem is

$$V(q_{k,f}) = \max\left\{V^a(q_{k,f}), V^r(k) = \frac{1}{1+\gamma}EV(q'_{k,f})\right\}$$

where $V(q_{k,f})$, $V^a(q_{k,f})$, and $V^r(k)$ denotes the value of having an offer $q_{k,f}$ in hand, the value of accepting a current offer, and the value of rejecting a current offer, respectively. $q'_{k,f}$ is the offer in the next period. Then, the solution will be

$$V(q_{k,f}) = \left\{\begin{array}{c} V^r(k) = \frac{1}{1+\gamma}EV(q'_{k,f}) \text{ if } q_{k,f} \leq q^{cut}_{k,f} \\ \\ V^a(q_{k,f}) \text{ if } q_{k,f} \geq q^{cut}_{k,f} \end{array}\right\}$$

where $q^{cut}_{k,f}$ is the cutoff quality with the property that a candidate should accept an offer $q_{k,f} \geq q^{cut}_{k,f}$ and reject an offer $q_{k,f} \leq q^{cut}_{k,f}$. More specifically, the value function of accepting this offer is given by

$$V^a(q_{k,f}) = y(q_{k,f}) + \frac{1}{1+\gamma}V^a(q_{k,f})$$

The flow value for a candidate who will serve as an outside director on boards with quality level $q_{k,f}$ denoted by $V^a(q_{k,f})$ equals the sum of the flow return, the reputation value from working as outside director $y(q_{k,f})$ plus the discounted expected flow value defined by $\frac{1}{1+\gamma}V^a(q_{k,f})$. We can rewrite above equation by

$$V^a(q_{k,f}) = \frac{(1+\gamma)y(q_{k,f})}{\gamma} \quad (1)$$

[5] Suppose there are n number of job openings and $n+1$ number of potential candidates. Then, the $n+1^{th}$ ranked candidate will get an offer only if at least one of the candidates ranked higher than him rejects an favorite offer. In general, we can not guarantee that all potential candidates will have received an offer.

[6] The reasons are that: (1) this model can generally guarantee that the first n number of upper ranked candidates will have received an offer and made a decision, and (2) I want to focus on the cutoff level of highly ranked candidates in the pool of potential candidates.

where the reputation function $y(q_{k,f})$ is increasing in $q_{k,f}$ and $y(0) = 0$. The value from rejection is defined by

$$V^r(k) = \frac{1}{1+\gamma} E_{prob^k, q'_{k,f}} \left[\max \left\{ V^a(q'_{k,f}), V^r(k) \right\} \right]$$

The value for a candidate of quality ranking k who waits for a better offer is denoted by $V^r(k)$ which equals the return from waiting and drawing again. Here, the expectation operation (E) is with respect to two components. One is the probability of successful matching with a better position denoted by $prob^k$ and the other is the value of the offer in the next period $q'_{k,f}$. In this model, all candidates have a well-ordered quality ranking and "take or leave it process" at each period mainly depends on their ranking, so that the probability of successful matching with a better position in a next period $prob^k$ plays a crucial role in determining the policy. More specifically, the value of the rejection is given by

$$V^r(k) = \frac{1}{1+\gamma} \left\{ \begin{array}{c} prob^k(q'_{k,f} \geq q^{cut}_{k,f}) E_{q'_{k,f}} [V^a(q'_{k,f})] \\ \\ + \left(1 - prob^k(q'_{k,f} \geq q^{cut}_{k,f})\right) V^r(k) \end{array} \right\} \tag{2}$$

where the expectation is only with respect to $q'_{k,f}$. The value for a candidate with quality ranking k who waits for a better offer is denoted by $V^r(k)$ equaling the discounted expected value from getting a better offer, $q'_{k,f} \geq q^{cut}_{k,f}$, in the future, which is defined by

$$\frac{1}{1+\gamma} prob^k(q'_{k,f} \geq q^{cut}_{k,f}) E_{q'_{k,f}} [V^a(q'_{k,f})]$$

plus the discounted expected value from rejection denoted by

$$\frac{1}{1+\gamma} \left(1 - prob^k(q'_{k,f} \geq q^{cut}_{k,f})\right) V^r(k)$$

$prob^k(q'_{k,f} \geq q^{cut}_{k,f})$ denotes the probability that the kth ranked candidate may get a better offer, $q'_{k,f} \geq q^{cut}_{k,f}$, in the next period. In other words, $prob^k(q'_{k,f} \geq q^{cut}_{k,f})$ represents the probability of successful matching with a better position than his cutoff quality. The probability function has the following property.

$$prob^k(q'_{k,f} \geq q^{cut}_{k,f}) = \left\{ \begin{array}{c} 0 \ if \ q^{cut}_{k,f} = \overline{q_f} \\ 1 \ if \ q^{cut}_{k,f} = 0 \end{array} \right\}$$

We can rewirite the equation (2) by

$$\left(r + prob^k(q'_{k,f} \geq q^{cut}_{k,f})\right) V^r(k) = \left\{ prob^k(q'_{k,f} \geq q^{cut}_{k,f}) E_{q'_{k,f}} [V^a(q'_{k,f})] \right\}$$

Finally, we can get the value of the rejection by

$$V^r(k) = \frac{(1+\gamma)prob^k(q'_{k,f} \geq q^{cut}_{k,f})E_{q'_{k,f}}[V^a(q'_{k,f})]}{\left(r + prob^k(q'_{k,f} \geq q^{cut}_{k,f})\right)} \tag{3}$$

Henceforth, the endogenous cutoff quality level denoted by $q^{cut}_{k,f}$ satisfies the following condition.

$$; \frac{y(q^{cut}_{k,f})}{\gamma} = \frac{prob^k(q'_{k,f} \geq q^{cut}_{k,f})E_{q'_{k,f}}[V^a(q'_{k,f})]}{\left(r + prob^k(q'_{k,f} \geq q^{cut}_{k,f})\right)}$$

Rewritting the above equation, we can get

$$y(q^{cut}_{k,f}) = \frac{\gamma}{\left(r + prob^k(q'_{k,f} \geq q^{cut}_{k,f})\right)} prob^k(q'_{k,f} \geq q^{cut}_{k,f})E_{q'_{k,f}}[V^a(q'_{k,f})] \tag{4}$$

The left-hand side is the cost of searching one more time when he got an offer, $q^{cut}_{k,f}$, and the right-hand side is the expected benefit of searching one more time. The equation (4) makes the candidate to set $q^{cut}_{k,f}$ to equate the cost and benefit of searching one more time.

Proposition 1 *Let me define the right hand side of equation (4) as*

$$B(q^{cut}_{k,f}) \equiv \frac{\gamma}{\left(r + prob^k(q'_{k,f} \geq q^{cut}_{k,f})\right)} prob^k(q'_{k,f} \geq q^{cut}_{k,f})E_{q'_{k,f}}[V^a(q'_{k,f})]$$

If $B(0) > 0$ *and* $\frac{\partial B(q^{cut}_{k,f})}{\partial q^{cut}_{k,f}} < 0$ *on* $(0, \overline{q_f})$*, there exists an unique cutoff level,* $q^{cut}_{k,f}$*,* $0 < q^{cut}_{k,f} < \overline{q_f}$*, which guarantees that the kth ranked candidate accepts an offer,* $q_{k,f}$*, if* $q_{k,f} \geq q^{cut}_{k,f}$*. Otherwise, he waits for a better offer in the future*

Proof. *See Appendix* ■

First, I focus on the cutoff level of the 1st ranked candidate, $q^{cut}_{1,f}$. If at least one job opening out of n number of job openings is drawn from the space above $q^{cut}_{1,f}$, the 1*st* ranked candidate gets a better offer than his cutoff level in the next period. The probability that the 1*st* ranked candidate can get a better offer $q^{cut}_{1,f} < q'_{1,f} \leq \overline{q_f}$ is defined by

$$prob^1(q'_{1,f} > q^{cut}_{1,f}) = \sum_{t=1}^{n} C^n_t \left(1 - F(q^{cut}_{1,f})\right)^t F(q^{cut}_{1,f})^{n-t} = 1 - F(q^{cut}_{1,f})^n$$

where C^n_t represents the distinguishible permutations of n objects, t of one type and $n-t$ of another type (binomial coefficient). The corresponding expected value of a better offer is

$$E_{q'_{1,f}}[V^a(q'_{1,f})] = \int_{q^{cut}_{1,f}}^{\overline{q_f}} \frac{y(q'_{1,f})}{\gamma} f(q'_{1,f})dq'_{1,f}$$

Henceforth, his cutoff level, $q_{1,f}^{cut}$, satisfies the equation (4) as

$$\frac{y(q_{1,f}^{cut})}{\gamma} = \frac{\left(\sum_{t=1}^{n} C_t^n \left(1 - F(q_{1,f}^{cut})\right)^t F(q_{1,f}^{cut})^{n-t}\right) \int_{q_{1,f}^{cut}}^{\overline{q_f}} \frac{y(q'_{1,f})}{\gamma} f(q'_{1,f}) dq'_{1,f}}{r + \left(\sum_{t=1}^{n} C_t^n \left(1 - F(q_{1,f}^{cut})\right)^t F(q_{1,f}^{cut})^{n-t}\right)}$$

Then, we can have the following proposition.

Proposition 2 *The probability of a successful matching with a better position than $q_{1,f}^{cut}$ in the next period, $prob^1(q'_{1,f} > q_{1,f}^{cut})$, decreases in $q_{1,f}^{cut}$ and there exists an unique cutoff level, $q_{1,f}^{cut}$, which guarantees that the 1st ranked candidate accepts an offer, $q_{1,f}$, if $q_{1,f} \geq q_{1,f}^{cut}$. Otherwise, he waits for a better offer in the future.*

Proof. *See Appendix* ■

Similarly, the probability that the $2nd$ ranked candidate can get a better offer $q_{2,f}^{cut} < q'_{2,f} < \overline{q_f}$ in the next period is

$$prob^2(q'_{2,f} > q_{2,f}^{cut}) = \left\{ \begin{array}{c} C_1^n \left\{F(q_{1,f}^{cut}) - F(q_{2,f}^{cut})\right\} F(q_{2,f}^{cut})^{n-1} \\ \\ + \sum_{t=2}^{n} C_t^n \left(1 - F(q_{2,f}^{cut})\right)^t F(q_{2,f}^{cut})^{n-t} \; if \; 0 \leq q_{2,f}^{cut} < q_{1,f}^{cut} \\ \\ \sum_{t=2}^{n} C_t^n \left(1 - F(q_{2,f}^{cut})\right)^t F(q_{2,f}^{cut})^{n-t} \; if \; q_{1,f}^{cut} \leq q_{2,f}^{cut} < \overline{q_f} \end{array} \right\}$$

Suppose that his cutoff level is below the cutoff level of the $1st$ ranked candidate. If only one job opening is drawn from the space between $q_{2,f}^{cut}$ and $q_{1,f}^{cut}$, the $2nd$ ranked candidate gets this offer because the 1^{st} ranked candidate will reject this one. The probability and expected value of a better offer for this case is given by

$$C_1^n \left\{F(q_{1,f}^{cut}) - F(q_{2,f}^{cut})\right\} F(q_{2,f}^{cut})^{n-1} \; and \int_{q_{2,f}^{cut}}^{q_{1,f}^{cut}} \frac{y(q'_{2,f})}{\gamma} f(q'_{2,f}) dq'_{2,f}$$

Whatever his cutoff level is below or above the cutoff level of the $1st$ ranked candidate, it is clear that if at least two job openings are drawn above $q_{2,f}^{cut}$, the $2nd$ ranked candidate will get a better offer. The corresponding probability and expected value of a high offer is

$$\sum_{t=2}^{n} C_t^n \left(1 - F(q_{2,f}^{cut})\right)^t F(q_{2,f}^{cut})^{n-t} \; and \int_{q_{2,f}^{cut}}^{\overline{q_f}} \frac{y(q'_{2,f})}{\gamma} f(q'_{2,f}) dq'_{2,f}$$

Therefore, his cutoff level, $q_{2,f}^{cut}$, solves to

$$\frac{y(q_{2,f}^{cut})}{\gamma} = V^r(2)$$

where

$$V^r(2) = \left\{ \begin{array}{c} \frac{\left(C_1^n\left\{F(q_{1,f}^{cut}) - F(q_{2,f}^{cut})\right\}F(q_{2,f}^{cut})^{n-1}\right) * \int_{q_{2,f}^{cut}}^{q_{1,f}^{cut}} \frac{y(q'_{2,f})}{\gamma} f(q'_{2,f}) dq'_{2,f}}{\gamma + C_1^n\left\{F(q_{1,f}^{cut}) - F(q_{2,f}^{cut})\right\}F(q_{2,f}^{cut})^{n-1}} \\ + \frac{\left(\sum_{t=2}^{n} C_t^n\left(1 - F(q_{2,f}^{cut})\right)^t F(q_{2,f}^{cut})^{n-t}\right) * \int_{q_{2,f}^{cut}}^{\overline{q_f}} \frac{y(q'_{2,f})}{\gamma} f(q'_{2,f}) dq'_{2,f}}{\gamma + \sum_{t=2}^{n} C_t^n\left(1 - F(q_{2,f}^{cut})\right)^t F(q_{2,f}^{cut})^{n-t}} \\ when\ 0 \leq q_{2,f}^{cut} < q_{1,f}^{cut} \\ \frac{\left(\sum_{t=2}^{n} C_t^n\left(1 - F(q_{2,f}^{cut})\right)^t F(q_{2,f}^{cut})^{n-t}\right) * \int_{q_{2,f}^{cut}}^{\overline{q_f}} \frac{y(q'_{2,f})}{\gamma} f(q'_{2,f}) dq'_{2,f}}{\gamma + \sum_{t=2}^{n} C_t^n\left(1 - F(q_{2,f}^{cut})\right)^t F(q_{2,f}^{cut})^{n-t}} \\ when\ q_{1,f}^{cut} \leq q_{2,f}^{cut} < \overline{q_f} \end{array} \right\}$$

The proposition follows.

Proposition 3 *The probability of a successful matching with a better position than* $q_{2,f}^{cut}$*,* $prob^2(q'_{2,f} > q_{2,f}^{cut})$*, decreases in* $q_{2,f}^{cut}$ *and there exists an unique cutoff level,* $q_{2,f}^{cut}$*, which guarantees that the 2nd ranked candidate accepts an offer,* $q_{2,f}$*, if* $q_{2,f} \geq q_{2,f}^{cut}$*. Otherwise, he waits for a better offer in the future.*

Proof. *Omitted.* ■

3.1.1 The probability of successful matching

Below, I generalize the probability function[7] that the kth ranked candidate $(3 \leq k \leq n)$[8] can get a better offer than his cutoff level, $prob^k(q'_{k,f} \geq q_{k,f}^{cut})$, to find the cutoff quality level of the kth ranked candidate by a numerical method. The cutoff quality level of the kth ranked candidate $q_{k,f}^{cut}$ satisfies the equation (4) given by

$$y(q_{k,f}^{cut}) = \frac{\gamma}{\left(r + prob^k(q'_{k,f} \geq q_{k,f}^{cut})\right)} prob^k(q'_{k,f} \geq q_{k,f}^{cut}) E_{q'_{k,f}}[V^a(q'_{k,f})]$$

Case 1: $0 \leq q_{k,f}^{cut} < Min[q_{j,f}^{cut}]$ First, I consider the case that $0 \leq q_{k,f}^{cut} < Min[q_{j,f}^{cut}]$, where $Min[q_{j,f}^{cut}]$ is the minimum cutoff level among all higher ranked candidates' cutoff levels than kth ranked candidate, $j = 1, 2, ...k-1$. If all candidates with higher ranking than k have an unique cutoff level, then the probability that the kth ranked candidate

[7] You can see more details in Appendix.

[8] As I mentioned before, I will focus on the cutoff level of the first n number of candidates in the pool of potential candidates because we can generally guarantee that the first n number of candidates will surely have received an offer in this model.

can get a better offer $q'_{k,f} > q^{cut}_{k,f}$ is defined by

$$prob^k(q'_{k,f} > q^{cut}_{k,f}) = G^k_A(q^{cut}_{k,f}) + G^k_C(q^{cut}_{k,f}) + G^k_D(q^{cut}_{k,f})\ if\ 0 \leq q^{cut}_{k,f} < Min[q^{cut}_{j,f}] \qquad (5)$$

where

$$\begin{aligned} G^k_A(q^{cut}_{k,f}) &= \sum_{i=z}^{k-1}\sum_{t=z-1}^{k-i} C^n_{t,i-1}\left(F\left((z-1)^{th}Min[q^{cut}_{j,f}]\right) - F\left((z-2)^{th}Min[q^{cut}_{j,f}]\right)\right)^{i-1} \\ &\quad * \left(1 - F((z-1)^{th}Min[q^{cut}_{j,f}])\right)^t F(q^{cut}_{k,f})^{n-t-i+1} \\ &\quad + \sum_{t=z}^{k-1} C^n_t\left(F\left((z-1)^{th}Min[q^{cut}_{j,f}]\right) - F\left((z-2)^{th}Min[q^{cut}_{j,f}]\right)\right)^t F(q^{cut}_{k,f})^{n-t} \\ where\ z &= 2, ..., k-1 \end{aligned}$$

$$G^k_C(q_{k,f}) = C^n_{k-1}\left(F\left((k-1)^{th}Min[q^{cut}_{j,f}]\right) - F\left((k-2)^{th}Min[q^{cut}_{j,f}]\right)\right)^{k-1} F(q^{cut}_{k,f})^{n-k+1}$$

and

$$G^k_D(q^{cut}_{k,f}) = \sum_{t=k}^{n} C^n_t\left(1 - F(q^{cut}_{k,f})\right)^t F(q^{cut}_{k,f})^{n-t}$$

Notice in $G^k_A(q^{cut}_{k,f})$ and $G^k_C(q^{cut}_{k,f})$ that

$$0^{th}Min[q^{cut}_{j,f}] = q^{cut}_{k,f}$$

$$Min[q^{cut}_{j,f}] = 1^{st}Min[q^{cut}_{j,f}]$$

and

$$(k-1)^{th}Min[q^{cut}_{j,f}] = Max[q^{cut}_{j,f}]$$

where $Min[q^{cut}_{j,f}](= 1^{st}Min[q^{cut}_{j,f}])$ is the minimum cutoff level among all higher ranked candidates' cutoff levels than kth ranked candidate, $j = 1, 2, ...k-1$, and $(k-1)^{th}Min[q^{cut}_{j,f}]$ is the maximum cutoff level among all candidates. Also, $z^{th}Min[q^{cut}_{j,f}]$ is the z^{th} minimum cutoff level among $q^{cut}_{j,f}$, $j = 1, 2, ...k-1$.

The interpretations of $G^k_A(q^{cut}_{k,f})$, $G^k_C(q^{cut}_{k,f})$, and $G^k_D(q^{cut}_{k,f})$ Even though less than k job openings are drawn from the space above $q^{cut}_{k,f}$, there is the possibility that the k^{th} ranked candidate will get a better offer in the future. $G^k_A(q^{cut}_{k,f})$ and $G^k_C(q^{cut}_{k,f})$ show the probability that these cases would occur. For instance, if only one job is drawn from the space between $q^{cut}_{k,f}$ and $Min[q^{cut}_{j,f}]$ and other $n-1$ job are drawn from the space below $q^{cut}_{k,f}$, the k^{th} ranked candidate will get a better offer in the future because all candidates ranked higher than the k^{th} ranked one will reject this offer. Let me show the process explicitly. Only one job, q'_f, is drawn from the space between $q^{cut}_{k,f}$ and $Min[q^{cut}_{j,f}]$. Then, all candidates ranked higher than the k^{th} ranked one will reject q'_f because q'_f is below their cutoff level. Since q'_f is above the cutoff level of the k^{th} ranked one, he will accept

q'_f . In this sense, the k^{th} ranked candidate will get a better offer than his cutoff level in the next period. We do not need to take into account $n-1$ jobs drawn from the space below $q^{cut}_{k,f}$ because all the first k^{th} ranked candidates (the first ranked one~the k^{th} ranked one) will reject these offers. Suppose only two jobs are drawn from the space between $Min[q^{cut}_{j,f}]$ and $2^{nd}Min[q^{cut}_{j,f}]$ and other $n-2$ job are drawn from the space below $q^{cut}_{k,f}$. Then, he can also get a better offer than his cutoff level because all higher ranked candidates except the candidate who has $Min[q^{cut}_{j,f}]$ quality level will reject these two offers. There are many different cases which guarantee that the k^{th} ranked candidate will get a better offer in the future even though less than k job openings are drawn from the space above $q^{cut}_{k,f}$. More explanations are given in the appendix. It is straightforward that if at least k jobs are drawn from the space above $q^{cut}_{k,f}$, the k^{th} ranked candidate gets a better offer. $G^k_D(q^{cut}_{k,f})$ shows the probability that this case would occur. Now, I will show a more explicit form of the expected flow value of getting a better offer,

$$prob^k(q'_{k,f} > q^{cut}_{k,f})E_{q'_{k,f}}[V^a(q'_{k,f})],\ if\ 0 \leq q^{cut}_{k,f} < Min[q^{cut}_{j,f}]$$

in Equation (4) by

$$prob^k(q'_{k,f} \quad > \quad q^{cut}_{k,f})E_{q'_{k,f}}[V^a(q'_{k,f})] = G^k_A(q^{cut}_{k,f}) * E_A V^a(q'_{k,f}) + G^k_C(q^{cut}_{k,f}) * E_C V^a(q'_{k,f})$$

$$+G^k_D(q^{cut}_{k,f}) * E_D V^a(q'_{k,f})\ if\ 0 \leq q^{cut}_{k,f} < Min[q^{cut}_{j,f}]$$

where

$$E_A V^a(q'_{k,f}) = \int_{(z-2)^{th}Min[q^{cut}_{j,f}]}^{(z-1)^{th}Min[q^{cut}_{j,f}]} \frac{y(q'_{k,f})}{\gamma} f(q'_{k,f})dq'_{k,f},\ \ z = 2, ...k-1$$

$$E_C V^a(q'_{k,f}) = \int_{(k-2)^{th}Min[q^{cut}_{j,f}]}^{(k-1)^{th}Min[q^{cut}_{j,f}]} \frac{y(q'_{k,f})}{\gamma} f(q'_{k,f})dq'_{k,f}$$

and

$$E_D V^a(q'_{k,f}) = \int_{q_{k,f}}^{\overline{q}} \frac{y(q'_{k,f})}{\gamma} f(q'_{k,f})dq'_{k,f}$$

$E_A V^a(q'_{k,f})$, $E_C V^a(q'_{k,f})$ and $E_D V^a(q'_{k,f})$ represent the expected flow value of a successful matching with a higher quality position than $q^{cut}_{k,f}$, which are mapped with the expected range of $q'_{k,f}$, respectively.

Case 2: $Min[q^{cut}_{j,f}] \leq q^{cut}_{k,f} < \overline{q_f}$ Second, I focus on the case that $Min[q^{cut}_{j,f}] \leq q^{cut}_{k,f} < \overline{q_f}$,where $Min[q^{cut}_{j,f}]$ is the minimum cutoff level, $j = 1, 2, ...k-1$. By a similar method, the probability [9]that the kth ranked candidate ($k \geq 3$) can get a better offer, $q'_{k,f} > q^{cut}_{k,f}$,

[9]See Appendix for more details.

is defined by

$$prob^k(q'_{k,f} \quad > \quad q_{k,f}^{cut}) \tag{6}$$

$$= \left\{ \begin{array}{c} G_F^k(q_{k,f}^{cut}) + G_H^k(q_{k,f}^{cut}) \\ \\ when\ (z-2)^{th}Min[q_{j,f}^{cut}] \leq q_{k,f}^{cut} < (z-1)^{th}Min[q_{j,f}^{cut}],\ z=3,.....k-1 \\ \\ G_G^k(q_{k,f}^{cut}) + G_H^k(q_{k,f}^{cut}) \\ \\ when\ (k-2)^{th}Min[q_{j,f}^{cut}] \leq q_{k,f}^{cut} < (k-1)^{th}Min[q_{j,f}^{cut}] = Max[q_{j,f}^{cut}] \\ \\ G_H^k(q_{k,f}^{cut}),\ when\ q_{k,f}^{cut}\ \geq (k-1)^{th}Min[q_{j,f}^{cut}] = Max[q_{j,f}^{cut}] \end{array} \right\}$$

where

$$G_F^k(q_{k,f}^{cut}) = \left\{ \begin{array}{c} \sum_{i=z}^{k-1}\sum_{t=1}^{k-i} C_{t,i-1}^n \left(F\left((z-1)^{th}Min[q_{j,f}^{cut}]\right) - F(q_{k,f}^{cut})\right)^{i-1} \\ \\ *\left(1-F((z-1)^{th}Min[q_{j,f}^{cut}])\right)^t F(q_{k,f}^{cut})^{n-t-i+1} \\ \\ +\sum_{t=z-1}^{k-1} C_t^n \left(F((z)^{th}Min[q_{j,f}^{cut}]) - F(q_{k,f}^{cut})\right)^t F(q_{k,f}^{cut})^{n-t} \end{array} \right\}$$

$$G_G^k(q_{k,f}^{cut}) = C_{k-1}^n \left(F\left((k-1)^{th}Min[q_{j,f}^{cut}]\right) - F(q_{k,f}^{cut})\right)^{k-1} F(q_{k,f}^{cut})^{n-k+1}$$

and

$$G_H^k(q_{k,f}^{cut}) = \sum_{t=k}^{n} C_t^n \left(1-F(q_{k,f}^{cut})\right)^t F(q_{k,f}^{cut})^{n-t}$$

Notice in $G_F^k(q_{k,f}^{cut})$ and $G_G^k(q_{k,f}^{cut})$ that

$$0^{th}Min[q_{j,f}^{cut}] = q_{k,f}^{cut}$$

$$Min[q_{j,f}^{cut}] = 1^{st}Min[q_{j,f}^{cut}]$$

and

$$(k-1)^{th}Min[q_{j,f}^{cut}] = Max[q_{j,f}^{cut}]$$

where $Min[q_{j,f}^{cut}](= 1^{st}Min[q_{j,f}^{cut}])$ is the minimum cutoff level among all higher ranked candidates' cutoff levels than kth ranked candidate, $j = 1, 2, ...k-1$, and $(k-1)^{th}Min[q_{j,f}^{cut}]$ is the maximum cutoff level among all candidates. Also, $z^{th}Min[q_{j,f}^{cut}]$ is the z^{th} minimum cutoff level among $q_{j,f}^{cut}$, $j = 1, 2, ...k-1$. When $k = 3$, we do not have to take into account the first range which is $(z-2)^{th}Min[q_{j,f}^{cut}] \leq q_{k,f}^{cut} < (z-1)^{th}Min[q_{j,f}^{cut}]$ in the equation (6).

The interpretations of $G_F^k(q_{k,f}^{cut})$, $G_G^k(q_{k,f}^{cut})$, and $G_H^k(q_{k,f}^{cut})$ Suppose that the cutoff level of the kth ranked candidate is on $Min[q_{j,f}^{cut}] \leq q_{k,f}^{cut} < 2^{nd}Min[q_{j,f}^{cut}]$. Only two job openings are drawn from the space between $q_{k,f}^{cut}$ and $2^{nd}Min[q_{j,f}^{cut}]$ and another $n-2$ jobs are drawn from the space below $q_{k,f}^{cut}$. Then, he will get a better offer because all higher ranked candidates except the candidate who has $Min[q_{j,f}^{cut}]$ will reject these two offers. We can generalize these cases in the following manner. Even though less than k job openings are drawn from the space above $q_{k,f}^{cut}$, the kth ranked candidate $(k \geq 3)$ can get a better offer when at least two job are drawn from the space between $q_{k,f}^{cut}$ and $2^{nd}Min[q_{j,f}^{cut}]$. Also, suppose that the cutoff level of the kth ranked candidate is on $2^{nd}Min[q_{j,f}^{cut}] \leq q_{k,f}^{cut} < 3^{rd}Min[q_{j,f}^{cut}]$. Only three job openings are drawn from the space between $2^{nd}Min[q_{j,f}^{cut}]$ and $3^{rd}Min[q_{j,f}^{cut}]$ and other $n-3$ jobs are drawn from space below $q_{k,f}^{cut}$. Then, he will get a better offer because all higher ranked candidates except the candidates who have $Min[q_{j,f}^{cut}]$ or $2^{nd}Min[q_{j,f}^{cut}]$ will reject these three offers. Even though less than k job openings are drawn from the space above $q_{k,f}^{cut}$, the kth ranked candidate $(k \geq 3)$ can get a better offer when at least three jobs are drawn from the space between $2^{nd}Min[q_{j,f}^{cut}]$ and $3^{rd}Min[q_{j,f}^{cut}]$. We can iterate this process to find the probability function. Under the assumption of $Min[q_{j,f}^{cut}] \leq q_{k,f}^{cut}$, $G_F^k(q_{k,f}^{cut})$ and $G_G^k(q_{k,f}^{cut})$ show the probability that the kth ranked candidate $(k \geq 3)$ can get a better offer than his cutoff level in the next period even though less than k job openings are drawn from the space above $q_{k,f}^{cut}$. It is clear that at least k number of job openings are drawn from the space above $q_{k,f}^{cut}$, then he will get a better offer. $G_H^k(q_{k,f}^{cut})$ provides the probability function of this case. Similarly, we can find a more explicit form of the expected flow value of getting a better offer, $prob^k(q_{k,f}^{cut})EV^a(q_{k,f}^{high})$, $(k \geq 3)$ in Equation (4) by

$$prob^k(q'_{k,f} > q^{cut}_{k,f})EV^a(q'_{k,f})$$

$$= \left\{ \begin{array}{c} G^k_F(q^{cut}_{k,f}) * \left\{ \int_{q^{cut}_{k,f}}^{(z-2)^{th}Min[q^{cut}_{j,f}]} \frac{y(q'_{k,f})}{\gamma} f(q'_{k,f})dq'_{k,f} \right\} \\ +G^k_H(q^{cut}_{k,f}) * \left\{ \int_{q^{cut}_{k,f}}^{\overline{q}} \frac{y(q'_{k,f})}{\gamma} f(q'_{k,f})dq'_{k,f} \right\} \\ when\ (z-1)^{th}Min[q^{cut}_{j,f}] \leq q^{cut}_{k,f} < (z-2)^{th}Min[q^{cut}_{j,f}],\ z = 3, ...k-1 \\ G^k_G(q^{cut}_{k,f}) * \left\{ \int_{q^{cut}_{k,f}}^{(k-1)^{th}Min[q^{cut}_{j,f}]} \frac{y(q'_{k,f})}{\gamma} f(q'_{k,f})dq'_{k,f} \right\} \\ +G^k_H(q^{cut}_{k,f}) * \left\{ \int_{q^{cut}_{k,f}}^{\overline{q}} \frac{y(q'_{k,f})}{\gamma} f(q'_{k,f})dq'_{k,f} \right\} \\ when\ (k-2)^{th}Min[q^{cut}_{j,f}] \leq q^{cut}_{k,f} < (k-1)^{th}Min[q^{cut}_{j,f}] = Max[q^{cut}_{j,f}] \\ G^k_H(q^{cut}_{k,f}) * \left\{ \int_{q^{cut}_{k,f}}^{\overline{q}} \frac{y(q'_{k,f})}{\gamma} f(q'_{k,f})dq'_{k,f} \right\} \\ when\ q^{cut}_{k,f}\ \geq (k-1)^{th}Min[q^{cut}_{j,f}] = Max[q^{cut}_{j,f}] \end{array} \right\}$$

4 Numerical Analysis

4.1 The quality of a firm: an uniform distribution

Here, I propose a numerical analysis. As I mentioned above, the cutoff quality levels satisfy the equation (4) which is given by

$$y(q^{cut}_{k,f}) = \frac{\gamma}{\left(\gamma + prob^k(q'_{k,f} \geq q^{cut}_{k,f})\right)} prob^k(q'_{k,f} \geq q^{cut}_{k,f})E_{q'_{k,f}}[V^a(q'_{k,f})]$$

For instance, the cutoff level of the first ranked candidiate solves to

$$y(q^{cut}_{k,f}) = \frac{\gamma\left(\sum_{t=1}^{n} C^n_t \left(1 - F(q^{cut}_{1,f})\right)^t F(q^{cut}_{1,f})^{n-t}\right) \int_{q^{cut}_{1,f}}^{\overline{q_f}} \frac{y(q'_{1,f})}{\gamma} f(q'_{1,f})dq'_{1,f}}{\gamma + \left(\sum_{t=1}^{n} C^n_t \left(1 - F(q^{cut}_{1,f})\right)^t F(q^{cut}_{1,f})^{n-t}\right)}$$

To calibrate the cutoff levels, we need the functional form for the reputation value generated by outside directorships $y(\cdot)$ and the distribution of firm quality $F(\cdot)$. Also, we need the value for a discount rate, γ and for the number of job openings at a given time, n. *Table 3* shows the functional form and the baseline value for γ and n.

Table 3 the functional form and the baseline value

$y(q_f)$ *:the reputation value generated by outside directorships*	$y(q_f) = q_f$
$F(q_f)$: *the distribution of firm quality*	*uniform distribution on [0,1]*
γ : *discount rate*	*0.05*
n : *number of job openings at a given time*	*30*

I assume that the reputational value function, $y(q_f)$, is linear in the quality of a firm and the distribution of firms' quality, $F(q_f)$, follows an uniform distribution on [0, 1]. Here, we can interpret the quality of a firm as the ranking of a firm in the economy. The results of calibrations are shown in *Table 4 and Figure 1.*

Table 4 the cutoff firm quality level of candidates for outside director with an uniform distribution

Ranking of candidates	*Cutoff level : firm ranking in percentage*	*Cutoff level: firm ranking (Given N=500), N: Total number of firm in the economy*
1st	*Lower 40%*	*300*
2nd	*Lower 40%*	*300*
3rd	*Lower 40%*	*300*
4th	*Lower 40%*	*300*
5th	*Lower 40%*	*300*
. . .		
10th	*≈Lower 40% (39.9988%)*	*≈300*
11th	*≈Lower 40% (39.9961%)*	*≈300*
12th	*≈Lower 40% (39.9885%)*	*≈300*
. . .		
20th	*Lower 36.61%*	*≈315*
. . .		
25th	*Lower 24.98%*	*≈375*

Figure 1 the cutoff firm quality level of candidates for outside director

- *C^k denotes the kth ranked candidate's cost function of one more searching*
- *B^k denotes the kth ranked candidate's expected benefit of one more searching*
- *The intersections represent the cutoff level of candidates*

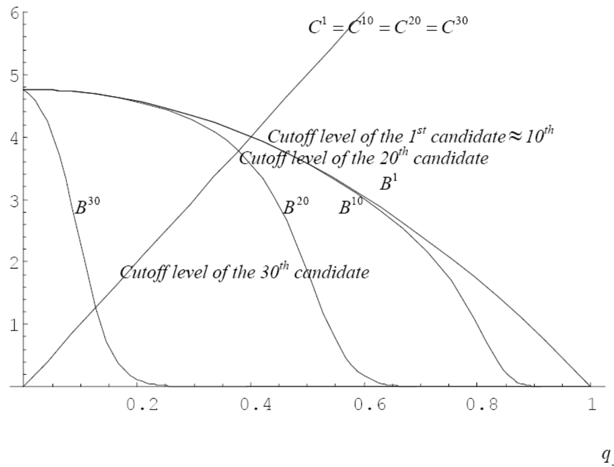

Table 4 shows the cutoff quality level of candidates for outside directors based on the functional forms and baseline value given in *Table 3*. The cutoff value of the first-ranked candidate is 0.4 (lower 40%). If there are 500 firms in the economy, this value represents the 300th ranked firm out of 500 firms. The cutoff-levels of high-ability candidates (say, 1st~12th candidates) are approximately the same because the difference in the probability of a successful matching with a better offer in the future among high ranked candidates is arbitrarily small in the middle and low range of firms' quality q_f. From the viewpoints of highly ranked candidates, if the quality of a current offer is relatively low, they expect that they would get a better offer in the future with the probability almost close to 1. *Figure 2* shows the probability of successful matching in the future by the rank of candidates.

Figure 2 the probability of successful matching in the future by the rank of candidates

- *Given a current offer, q_f, each graph represents the probability of successful matching with a better offer in the future*
- *k denotes the ranking of candidates*

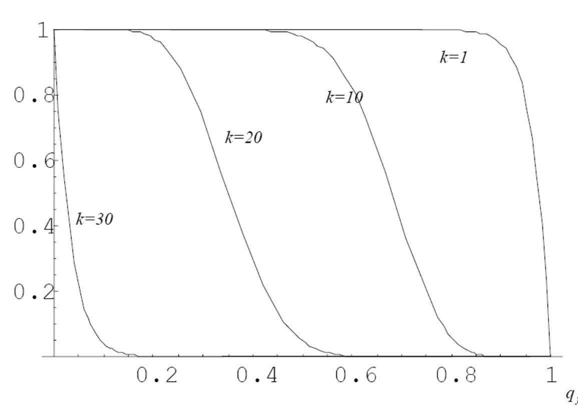

Table 5-A shows the sensitivity of the cutoff level to the change in a discounted rate, γ. In the case of the first-ranked candidate, the cutoff level decreases in a discounted rate. The interpretation is clear. As the candidate becomes less patient, his cutoff level decreases. *Table 5-B* provides the effect of a change in the functional form of reputation value generated by outside directorships on the cutoff level of the 1st ranked candidate. When $y(q_f) = q_f^2$, the cutoff level is the 240 ranked firm over 500 firms in the economy, which is compared to the 300 ranked firm in the linear form, $y(q_f) = q_f$. Since the marginal reputation value increases in q_f under the functional form, $y(q_f) = q_f^2$, the highly-ranked candidates would be more likely to wait for a better offer in the next period.

Table 5-A the sensitivity analysis: the effect of change in a discounted rate on the cutoff level of the 1st ranked candidate for outside director

γ : discount rate	Cutoff level : firm ranking in percentage	Cutoff level: firm ranking (Given N=500)
0.05	≈Lower 40%	≈300
0.1	≈Lower 38.7%	≈306
0.15	≈Lower 37.4%	≈314
0.2	≈Lower 36.2%	≈320
0.25	≈Lower 35.1%	≈324
0.3	≈Lower 34.01%	≈330

Table 5-B the sensitivity analysis: the effect of change in the function form $y(q_k)$ on the cutoff level of the 1st ranked candidate for outside director

$y(q_k)$: reputation value	Cutoff level : firm ranking in percentage	Cutoff level: firm ranking (Given N=500)
q_k^2	Lower 52.2%	≈240

4.2 The quality of a firm: an extreme value distribution

Here, I assume that the quality of a firm, $F(q_f)$, follows the extreme value distribution infered by Gabaix and Landier (2008).[10]

[10] Gabaix and Landier (2008) show that the density of talent t, left of the upper bound $T_{\max}$, is

$$f(t) = \frac{3B'}{2}(T_{\max} - T)^{\frac{1}{2}}, \text{for } t \text{ close to } T_{\max}$$

so, I approximate the upper tail distribution of firms' quality $F(q_f)$ by

$$F(q_f) = -B'(\overline{q_f} - q_f)^{\frac{1}{2}} + 1 \text{ on } [0, \overline{q_f}],\ F(0) = 0 \text{ and } F(\overline{q_f}) = 1$$

Table 6 the cutoff level of candidate for outside director with an extreme value distribution

- $F(q_k)$ *follows the distribution of CEO talent inferred by Gabaix and Landier (2006):* $F(q_k) = -B'(\overline{q_f} - q_f)^{\frac{1}{2}} + 1$, on $[0, \overline{q_f}]$

Ranking of candidates	*Cutoff level : talent level of CEO in percentage*	*Cutoff level: firm ranking (Given N=500), N: Total number of firm in the economy*
1st	*≈Lower 44% (43.58%)*	*≈280*
2nd	*≈Lower 44% (43.58%)*	*≈280*
3rd	*≈Lower 44% (43.58%)*	*≈280*
4th	*≈Lower 44% (43.58%)*	*≈280*
5th	*≈Lower 44% (43.58%)*	*≈280*
. . .		
10th	*≈Lower 44% (43.58%)*	*≈280*
. . .		
20th	*≈Lower 40% (40.03%)*	*≈300*
. . .		
25th	*≈Lower 30% (30.41%)*	*≈350*

Table 6 provides the cutoff level of candidates for outside directors by the ranking under the assumption that $F(q_f)$ follows an extreme value distribution.In this case, I also use the linear form of the reputational value, $y(q_f)$, and the baseline value for γ and n. If there are 500 firms in the economy, the cutoff level of the first ranked candidate is the 280th best of 500 firms. Overall, the high-ability candidates (say, 1st~25th candidates) would be willing to accept an offer from lower 30%~44% firms. (280th~350th best of 500 firms)

5 Conclusion

I construct a matching model ("take it or leave it offer") in a frictional directorship market to explain the (observed) unbalanced match between the quality of outside directors on boards and the firms. My calibration shows that if the firm's quality follows an uniform distribution, the best candidate would be willing to accept an offer from the lower 25% firm (e.g., 300th best of 500 firms), and the high-ability candidates (say, the 1st~25th ranked ones in my calibration) would be willing to accept an offer from the lower 25%~40% firms. Also, the best candidate would be likely to accept an offer from the lower 30% firm, and the high-ability candidates (say, the 1st~25th ranked ones in my calibration) from the lower 30%~46% firms under the extreme value distribution. This calibration could explain the observed facts that the highly qualified candidates (the 1st~25th ranked directors, upper 0%~5% in my sample) are sometimes matched with lower 25%~49% ranked firms, and, more broadly, the 1st~50th ranked directors (upper 0%~10% in my sample) with lower 20%~49% firms.

References

Brickley, James A., James S. Linck and Jeffrey L. Coles. 1999. "What Happens to CEOs After They Retire? New Evidence On Career Concern, Horizon Problem, and CEO Incentives." *Journal of Financial Economics* 52:341–377.

Colley, John L., Jacqueline L. Doyle George W. Logan and Wallace Stettinius. 2003. *Corporate Governance*. New-York: Mcgraw-Hill.

Gabaix, Xavier and Augustin Landier. 2008. "Why Has CEO Pay Increased So Much?" *Quarterly Journal of Economics* 123:49–100.

Gale, David and L. S. Shapley. 1962. "College Admissions and the Stability of Marriage." *The American Mathematical Monthly* 69:9–15.

Hambrick, Donald C and Eric M. Johnson. 2000. "Outsider directors with a stake: The Linchpin in improving governance." *California Management Review* 42:108–127.

Lee, Changmin. 2007b. "The Market for Corporate Directors." *CAEPR Working Paper* available at SSRN: http://ssrn.com/abstract=988119(2007-006).

Lee, Changmin. 2007c. "What happens to retired CEOs now? A Re-Examination of Brickley et al.s 'What happens to retired CEO.s?'." *CAEPR Working Paper* available at SSRN: http://ssrn.com/abstract=988162(2007-007).

McCall, John. 1970. "Economics of Information and Job Search." *Quartely Journal of Economics* 84(1):113–126.

6 Appendix I

Proof. of the Proposition 1: By assumption, $y(0) = 0$, $y(\overline{q_f}) > 0$, and $y(q^{cut}_{k,f})$ is increasing in $q^{cut}_{k,f}$, so there exists an unique cutoff level. ■

Proof. of the Proposition 2: (1) $B(0) = \int_0^{\overline{q_f}} \frac{y(q'_{1,f})}{\gamma} f(q'_{1,f}) dq'_{1,f} > 0$ and (2)

$$\frac{\partial B(q^{cut}_{1,f})}{\partial q^{cut}_{1,f}} = \frac{\partial \left\{ r \frac{prob^1(q'_{1,f} > q^{cut}_{1,f}) \int_{q^{cut}_{1,f}}^{\overline{q_f}} \frac{y(q'_{1,f})}{\gamma} f(q'_{1,f}) dq'_{1,f}}{r + \left(prob^1(q'_{1,f} > q^{cut}_{1,f})\right)} \right\}}{\partial q^{cut}_{1,f}}$$

$$= r \frac{\left\{ \frac{\partial prob^1(q'_{1,f} > q^{cut}_{1,f})}{\partial q^{cut}_{1,f}} \left(\int_{q^{cut}_{1,f}}^{\overline{q_f}} \frac{y(q'_{1,f})}{\gamma} f(q'_{1,f}) dq'_{1,f} \right) + prob^1(q'_{1,f} > q^{cut}_{1,f}) \left(-\frac{y(q'_{1,f})}{\gamma} f(q'_{1,f}) \right) \right\} \left\{ r + prob^1(q'_{1,f} > q^{cut}_{1,f}) \right\}}{\left\{ r + prob^1(q'_{1,f} > q^{cut}_{1,f}) \right\}^2}$$

$$-r \frac{prob^1(q'_{1,f} > q^{cut}_{1,f}) \int_{q_{1,f}}^{\overline{q_f}} \frac{y(q'_{1,f})}{\gamma} f(q'_{1,f}) dq'_{1,f} \left(\frac{\partial prob^1(q'_{1,f} > q^{cut}_{1,f})}{\partial q^{cut}_{1,f}} \right)}{\left\{ r + prob^1(q'_{1,f} > q^{cut}_{1,f}) \right\}^2}$$

$$= r \frac{\left\{ \frac{\partial prob^1(q'_{1,f} > q^{cut}_{1,f})}{\partial q^{cut}_{1,f}} \left(\int_{q^{cut}_{1,f}}^{\overline{q_f}} \frac{y(q'_{1,f})}{\gamma} f(q'_{1,f}) dq'_{1,f} \right) + prob^1(q'_{1,f} > q^{cut}_{1,f}) \left(-\frac{y(q'_{1,f})}{\gamma} f(q'_{1,f}) \right) \right\} r}{\left\{ r + prob^1(q'_{1,f} > q^{cut}_{1,f}) \right\}^2}$$

$$+r \frac{\left\{ prob^1(q'_{1,f} > q^{cut}_{1,f}) \left(-\frac{y(q'_{1,f})}{\gamma} f(q'_{1,f}) \right) \right\} prob^1(q'_{1,f} > q^{cut}_{1,f})}{\left\{ r + prob^1(q'_{1,f} > q^{cut}_{1,f}) \right\}^2} \leq 0$$

Then, the proposition 1 is satisfied, which implies that there exists a unique cutoff level. ■

7 Appendix II: The probability that the kth ranked candidate can get a better offer, $prob^k(q'_{k,f} > q^{cut}_{k,f})$

Here, I show how to derive the probability that the kth ranked candidate can get a better offer, $q'_{k,f} > q^{cut}_{k,f}$. Let me explain the outline by showing the case of the 5^{th} ranked candidate.

Case 1

1. Suppose that his cutoff level, $q^{cut}_{5,f}$, is less than the minimum cutoff level of all higher ranked candidates, $q^{cut}_{5,f} < Min[q^{cut}_{1,f}, q^{cut}_{2,f}, q^{cut}_{3,f}, q^{cut}_{4,f}]$. Then, if at least five job openings out of n number job openings are drawn from the space above $q^{cut}_{5,f}$, he

will get a better offer in the future. The probability of this case is that

$$prob_a^5(q_{5,f}^{cut}) = \sum_{t=5}^{n} C_t^n \left(1 - F(q_{5,f}^{cut})\right)^t F(q_{5,f}^{cut})^{n-t}$$

$$where\ C_t^n = \begin{pmatrix} n \\ t \end{pmatrix}$$

where C_t^n represents the distinguishible permutations of n objects, t of one type and $n-t$ of another type (binomial coefficient).

2. From here, suppose that less than 5 jobs are drawn from the space above $q_{5,f}^{cut}$. If only four job openings out of n number job openings are drawn form the space between $3^{rd}Min[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}]$ and $4^{th}Min[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}] = Max[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}]$, he will get a better offer because the candidate who has the maximum cutoff level will reject all offers. The probability is

$$prob_b^5(q_{5,f}^{cut}) = C_4^n \left(F(Max) - F(3^{rd}Min)\right)^4 F(q_{5,f}^{cut})^{n-4}$$

3. If three or four job openings are drawn from the space between $2^{nd}Min[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}]$ and $3^{rd}Min[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}]$, then he can also get a better offer with the probability

$$\begin{aligned} prob_c^5(q_{5,f}^{cut}) &= C_3^n \left(F(3^{rd}Min) - F(2^{nd}Min)\right)^3 F(q_{5,f}^{cut})^{n-3} \\ &+ C_4^n \left(F(3^{rd}Min) - F(2^{nd}Min)\right)^4 F(q_{5,f}^{cut})^{n-4} \\ &+ C_{3,1}^n \left(F(3^{rd}Min) - F(2^{nd}Min)\right)^3 * \left(F(\overline{q_f}) - F(3^{rd}Min)\right)^1 F(q_{5,f}^{cut})^{n-4} \\ where\ C_{a,b}^n &= \begin{pmatrix} n \\ a, b, n-a-b \end{pmatrix} \end{aligned}$$

because both candidates who have the maximum cutoff level and $3^{rd}Min[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}]$ will reject all offers in the case of the first two terms in the probability function and one of them will reject all offers in the final case. $C_{a,b}^n$ represents the distinguishible permutations of n objects, a of one type, b of second type and $n-a-b$ of third type (multinomial coefficient).

4. If two, three or four job openings are drawn from the space between $Min[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}]$ $=1^{st}Min[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}]$ and $2^{nd}Min[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}]$, then he can also get

a better offer with the probability

$$\begin{aligned}
prob_d^5(q_{5,f}^{cut}) &= C_2^n \left(F(2^{nd}Min) - F(Min)\right)^2 F(q_{5,f}^{cut})^{n-2} \\
&+ C_3^n \left(F(2^{nd}Min) - F(Min)\right)^3 F(q_{5,f}^{cut})^{n-3} \\
&+ C_4^n \left(F(2^{nd}Min) - F(Min)\right)^4 F(q_{5,f}^{cut})^{n-4} \\
&+ C_{2,1}^n \left(F(2^{nd}Min) - F(Min)\right)^2 * \left(F(\overline{q_f}) - F(2^{nd}Min)\right)^1 F(q_{5,f}^{cut})^{n-3} \\
&+ C_{2,2}^n \left(F(2^{nd}Min) - F(Min)\right)^2 * \left(F(\overline{q_f}) - F(2^{nd}Min)\right)^2 F(q_{5,f}^{cut})^{n-4}
\end{aligned}$$

5. Finally, if one, two, three or four job opening are drawn from the space between $q_{5,f}^{cut}$ and $Min[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}]$ $=1^{st}Min[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}]$, then he also can get a better offer with the probability

$$\begin{aligned}
prob_g^5(q_{5,f}^{cut}) &= C_1^n \left(F(Min) - F(q_{5,f}^{cut})\right)^1 F(q_{5,f}^{cut})^{n-1} \\
&+ C_2^n \left(F(Min) - F(q_{5,f}^{cut})\right)^2 F(q_{5,f}^{cut})^{n-2} \\
&+ C_3^n \left(F(Min) - F(q_{5,f}^{cut})\right)^3 F(q_{5,f}^{cut})^{n-3} \\
&+ C_4^n \left(F(Min) - F(q_{5,f}^{cut})\right)^4 F(q_{5,f}^{cut})^{n-4} \\
&+ C_{1,1}^n \left(F(Min) - F(q_{5,f}^{cut})\right)^1 * \left(F(\overline{q_f}) - F(Min)\right)^1 F(q_{5,f}^{cut})^{n-2} \\
&+ C_{1,2}^n \left(F(Min) - F(q_{5,f}^{cut})\right)^1 * \left(F(\overline{q_f}) - F(2^{nd}Min)\right)^2 F(q_{5,f}^{cut})^{n-3} \\
&+ C_{1,3}^n \left(F(Min) - F(q_{5,f}^{cut})\right)^1 * \left(F(\overline{q_f}) - F(Min)\right)^3 F(q_{5,f}^{cut})^{n-4}
\end{aligned}$$

Case 2

1. Suppose that $q_{5,f}^{cut}$ is on $Min[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}] \leq q_{5,f}^{cut} < 2^{nd}Min[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}]$ and less than k job openings are drawn from the space above $q_{5,f}^{cut}$. Then, if two, three or four job openings out of n number job openings are drawn from the space between his current offer and $2^{nd}Min[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}]$, he will get a better offer.

The probability is that

$$
\begin{aligned}
prob_h^5(q_{5,f}^{cut}) \quad = \quad & C_2^n \left(F(2^{nd}Min) - F(q_{5,f}^{cut})\right)^2 F(q_{5,f}^{cut})^{n-2} \\
& + C_3^n \left(F(2^{nd}Min) - F(q_{5,f}^{cut})\right)^3 F(q_{5,f}^{cut})^{n-3} \\
& + C_4^n \left(F(2^{nd}Min) - F(q_{5,f}^{cut})\right)^4 F(q_{5,f}^{cut})^{n-4} \\
& + C_{2,1}^n \left(F(2^{nd}Min) - F(q_{5,f}^{cut})\right)^2 * \left(F(\overline{q_f}) - F(2^{nd}Min)\right)^1 F(q_{5,f}^{cut})^{n-3} \\
& + C_{2,2}^n \left(F(2^{nd}Min) - F(q_{5,f}^{cut})\right)^2 * \left(F(\overline{q_f}) - F(2^{nd}Min)\right)^2 F(q_{5,f}^{cut})^{n-4}
\end{aligned}
$$

2. Also, if at least five job openings out of n number job openings are drawn from the space above $q_{5,f}^{cut}$, he will get a better offer in the future. The probability is that

$$
prob_i^5(q_{5,f}) = \sum_{t=5}^{n} C_t^n \left(1 - F(q_{5,f}^{cut})\right)^t F(q_{5,f}^{cut})^{n-t}
$$

Case 3

1. Also, suppose that less than 5 job openings are drawn from the space above $q_{5,f}^{cut}$. In this case, if three or four job openings out of n number job openings are drawn from the space between his current offer and $3^{rd}Min[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}]$ he will get a better offer with probability

$$
\begin{aligned}
prob_j^5(q_{5,f}) \quad = \quad & C_3^n \left(F(3^{rd}Min) - F(q_{5,f}^{cut})\right)^3 F(q_{5,f}^{cut})^{n-3} \\
& + C_4^n \left(F(3^{rd}Min) - F(q_{5,f}^{cut})\right)^4 F(q_{5,f}^{cut})^{n-4} \\
& + C_{3,1}^n \left(F(3^{rd}Min) - F(q_{5,f}^{cut})\right)^3 * \left(F(\overline{q_f}) - F(3^{rd}Min)\right)^1 F(q_{5,f}^{cut})^{n-4}
\end{aligned}
$$

2. Also, if at least five (k) job openings out of n number job openings are drawn from the space above $q_{5,f}^{cut}$, he will get a better offer in the future. The probability is that

$$
prob_i^5(q_{5,f}^{cut}) = \sum_{t=5}^{n} C_t^n \left(1 - F(q_{5,f}^{cut})\right)^t F(q_{5,f}^{cut})^{n-t}
$$

Case 4

1. Also, suppose that less than 5 job openings are drawn from the space above $q_{5,f}^{cut}$. If

four job openings out of n number job openings are drawn from the space between his cutoff level and $Max[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}]$, he will get a better offer with probability

$$prob_l^5(q_{5,f}^{cut}) = C_4^n \left(F(Max) - F(q_{5,f}^{cut})\right)^4 F(q_{5,f}^{cut})^{n-4}$$

2. Also, if at least five job openings out of n number job openings are drawn from the space above $q_{5,f}^{cut}$, he will get a better offer in the future. The probability is that

$$prob_i^5(q_{5,f}^{cut}) = \sum_{t=5}^{n} C_t^n \left(1 - F(q_{5,f}^{cut})\right)^t F(q_{5,f}^{cut})^{n-t}$$

Case 5 Suppose that $Max[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}] \le q_{5,f}^{cut} < \overline{q_f}$. In this case, he will get a better offer in the future only if at least five job openings out of n number job openings are drawn from the space above $q_{5,f}^{cut}$. It is given by

$$prob_i^5(q_{5,f}^{cut}) = \sum_{t=5}^{n} C_t^n \left(1 - F(q_{5,f}^{cut})\right)^t F(q_{5,f})^{n-t}$$

Overall, the probability that the 5^{th} ranked candidate can get a better offer, $q_{5,f}' \ge q_{5,f}^{cut}$, in the future is

$$prob^5(q_{5,f}^{cut}) = \left\{ \begin{array}{c} prob_a^5(q_{5,f}^{cut}) + prob_b^5(q_{5,f}^{cut}) + prob_c^5(q_{5,f}^{cut}) + prob_d^5(q_{5,f}^{cut}) + prob_g^5(q_{5,f}^{cut}) \\ when\ q_{5,f}^{cut} < Min[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}] \\ prob_h^5(q_{5,f}^{cut}) + prob_i^5(q_{5,f}^{cut}) \\ when\ Min[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}] \le q_{5,f}^{cut} < 2^{nd}Min[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}] \\ prob_j^5(q_{5,f}^{cut}) + prob_i^5(q_{5,f}^{cut}) \\ 2^{nd}Min[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}] \le q_{5,f}^{cut} < 3^{rd}Min[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}] \\ prob_l^5(q_{5,f}^{cut}) + prob_i^5(q_{5,f}^{cut}) \\ 3^{rd}Min[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}] \le q_{5,f}^{cut} < 4^{th}Min[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}] = Max[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}] \\ prob_i^5(q_{5,f}^{cut})\ when\ Max[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}] \le q_{5,f}^{cut} < \overline{q_f} \end{array} \right\}$$