Public Sector Pension Policies and Capital Accumulation in Emerging Economies*

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Abstract

In emerging economies pension programs of public sector workers tend to be more generous than pension programs of private sector workers. In this paper we study the adverse effects of these generous pension schemes on income and welfare, using a two-sector overlapping generations model. We argue that opportunity costs of running generous public pension schemes for civil servants are potentially large in emerging economies, where there is often severe lack of public investments in education and infrastructure. Calculating transitions to the post reform steady state, we find that welfare gains for the generation born before the reform can only be realized when freed up resources are reinvested into public education public, but not when these freed up resources are used for tax cuts or for infrastructure investments.

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1 Introduction

Pension programs for civil servants are on average more generous than pension programs for private sector workers. This is true for OECD countries as well as for emerging economies and developing countries. Palacios and Whitehouse (2006) report that OECD countries spend on average one quarter of total pension payments on public sector retirees, whereas in developing countries this share is much larger. Replacement rates for public sector workers tend to be considerable larger than the national average (see Table 2 in Palacios and Whitehouse (2006)). According to a recent OECD report on Brazil, public spending on pensions accounts for over 10 percent of GDP, a higher share than in the average OECD country, despite Brazil's younger population (OECD (2005)). A large share (almost one half) goes to public sector retirees (Souza et al. (2004)).

In the presence of population aging this generous policy seems problematic as it puts a heavy burden on the budget of any economy. This is especially true for emerging economies where the tax base is smaller and generous pension programs divert much needed resources away from alternative uses like infrastructure investments or public education.

There is very little justification for running two different pension schemes simultaneously. The argument that pension programs for civil servants have to be more generous in order to compensate civil servants for lower public wages only holds partly for emerging economies. There is evidence that the wage level in the public sector is actually higher than in the private sector (e.g. Foguel et al. (2000), Panizza (2000), Panizza (2001), and Panizza and Qiang (2005)). If on top of that public pension programs are more generous than private pension programs, equity issues will arise, in addition to concerns about economic sustainability in countries with a high income concentration and the beginnings of population aging. It is surprising that very little has been written on the reform of public sector pension programs compared to the voluminous literature on national pension programs.

In this paper we study the adverse effects of generous public sector pension policies. We identify at least three channels through which generous pension to civil servants distort the economy. First, generous pensions result in adverse effects on capital accumulation. Second, generous public pension schemes are costly to finance by distortive tax-financing instruments, which then again distort capital accumulation. Third, the forgone opportunities of investing these resources into other productive government activities can be substantial, especially in developing countries. The effects from the first two channels have been well documented in the literature on social security since the seminal contribution by Auerbach and Kotlikoff (1987). This literature concentrated on national social security systems. Few papers have concentrated on sector specific social security reform assuming that the small number of public sector retirees would only allow for small post reform effects. Glomm, Jung and Tran (2006) show that these adverse effects are substantial concentrating on early retirement issues. In this paper we concentrate on exploring the effects of the third channel, alternative investments, while also taking capital accumulation effects into account. We think that the previous literature has understated the potential efficiency gains from reducing the generosity of public sector pensions and using free up resources for investments into infrastructure and public education.

Analyzing the economic effects of generous public sector pensions requires a fully specified dynamic general equilibrium model in which recipients of public sector pensions, civil servants, play a meaningful economic role. We employ one such model in which civil servants work in two sub-sectors, public education and public provision of infrastructure. This set-up allows us to not only study the costs of public sector compensation including pension benefits but also the benefits of public sector employment. In addition, the government invests in a public capital to provide services to households and firms. These services are made available free of charge. We can think of these as being services flowing from the stock of roads and highways. The government also finances public expenditures on education and social security payments to the private sector workers. In our model financing generous public sector pensions implies the opportunity cost of lower public expenditures on public education and/or on public capital accumulation.

In order to obtain quantitative results we calibrate the model to Brazil where the public pension system is unusually generous. In the policy experiments conducted we first focus on steady state outcomes and then compute transitions. Decreasing the generosity of public sector pensions and increasing either public education expenditures or investment in public capital has sizeable effects on long-run income. We find that the direct effects of public pension reform through influencing savings are relatively small. However, we find that the effects of reducing distortive tax rates are non-trivial. In addition, we find that using the resources that become available from the reduction in public pensions payments on public investment in infrastructure or on public education has large effects on output and welfare. Welfare gains for the generation born before the reform can only be realized when the freed up resources are reinvested into public education, but not when these freed up resources are used for tax cuts or for infrastructure investments. These welfare results that differ by the alternative use of public funds have important implications for implementing social security reforms. Only when there is a majority of the currently alive that benefits from the reform, can we expect such reforms to be implemented. In our case, the long run gains from such reforms can only be expected to be realized when the decreased generosity of pensions for civil servants results in higher education expenditure by the government. We also conduct sensitivity analysis and find that our results are robust to changes in parameter values.

Public sector pension reform attempts have been marred by political opposition of the small but well organized group of civil servants. While the long run costs of very generous pensions and the long run gains from pension reform are clear and well documented in the literature, it is crucial to find a way to overcome short run political opposition. Our model clearly shows that if the decrease in public sector pensions is accompanied by tax cuts there will be substantial welfare losses in the current generation. One of the main reasons is that the current old who work in the private sector will see a decrease in interest income since the increase in capital accumulation will lower the interest rate on their savings. These welfare losses will most likely lead to political opposition and doom any attempts at meaningful reform. A key result of our paper is to show that this political opposition can be overcome if the released public funds are allocated towards more public education. A higher level of public education expenditure will increase human capital and wage income. This increase in wage income compensates the initial old in the private sector who are facing declining interest income because of the higher savings of civil servants. We are not aware of any other papers on public sector pension reforms that point out how the allocation of public funds towards public education rather than tax cuts make the cut in pensions more politically feasible.

The adverse effects of public pensions via forgone investment opportunities for other productive government activities are neglected in the literature on social security. Our key contribution is to show that these effects are potentially large. In addition, our positive analysis could be used as an important justification for reforming public pension systems in developing countries with a low level of public investments.

The following section describes the model and the definition of the competitive equilibrium in detail. In section 3 we calibrate the model to Brazil and in section 4 we conduct policy experiments and discuss the results. Section 5 concludes. In Appendix A we lay out our solution method of the model. Appendix B contains all tables and figures.

2 The Model

2.1 Environment

There is a large number of individuals who live for two periods in an overlapping generation set-up. Each period accounts for roughly 30 years. For reasons of simplicity we abstract from population growth and normalize the size of the population to one. A fraction N^p of the population is working in the private sector and N^g is the fraction of civil servants. Workers who work in the public sector but do not have the status of a civil servant are counted as private sector workers. Only civil servants have access to generous pension payments. We therefore get

$$N^p + N^g = 1.$$

We distinguish two groups among civil servants. A fraction N^{ge} of civil servants is working in the public education sector , the others N^{gi} are working in the "public infrastructure" sector. We use the following notation

$$\begin{aligned} N^{ge} &= aN^{g}, \\ N^{gi} &= (1-a)\, N^{g}. \end{aligned}$$

All civil servants have an identical wage and pension scheme regardless of sector of employment. This scheme differs from private sector workers in contribution rates and also in benefit payments.

Agents value two different types of goods, a privately provided good and a publicly provided good. The utility function of a member of generation t is

$$u\left(c_t, G_t, c_{t+1}, G_{t+1}\right) = \frac{1}{1-\sigma}\left[c_t^{1-\sigma} + \Theta G_t^{1-\sigma}\right] + \pi\beta\frac{1}{1-\sigma}\left[c_{t+1}^{1-\sigma} + \Theta G_{t+1}^{1-\sigma}\right],$$

where c_s is consumption of the private good, G_s is a pure public good provided by the government in the two respective periods $s = t, t+1$ and π is an exogenous survival probability. We can think of this public good as enforcement of private property, enforcement of contracts, maintenance of law and order. Alternatively, we can think of this good as roads, highways or other elements of core infrastructure which is made available to all households and firms at a zero price. We also assume $\sigma > 0$.

The privately supplied good is produced from three inputs, the publicly provided service G_t, the private capital stock K_t and effective labor (human capital) in the private sector $H_t^p = H_t N_t^p$ according to the production function

$$Y_t = AG_t^{\alpha_1} K_t^{\alpha_2} \left(H_t^p\right)^{\alpha_3},$$

where $\alpha_i \in (0,1)$ for $i = 1, 2, 3$, $\alpha_2 + \alpha_3 = 1$, and $A > 0$. Capital K fully depreciates each period. The public good G is provided without charge by the government. If G is made available to firms at a zero price, firms only hire capital and labor. The condition $\alpha_2 + \alpha_3 = 1$ then ensures constant returns to scale in the two hired factors and zero profits. This kind of production function is standard and has been used by Barro (1990), Glomm and Ravikumar (1994), Turnovsky and Fisher (1995), Cassou and Lansing (1998) and many others.

Human capital is produced according to

$$h_{t+1} = D\left[\left(H_t^{ge}\right)^{\eta_1} + \chi_1 E_t^{\eta_1}\right]^{\frac{\gamma_1}{\eta_1}} h_t^{\gamma_2}, \tag{1}$$

where H_t^{ge} is public educational human capital (teachers), E_t is public education expenditure, h_t is the parental human capital, $D > 0$, $\eta_1 \leq 1, (\gamma_1, \gamma_2) \in (0,1)$, and $\gamma_1 + \gamma_2 \leq 1$.

Most models of human capital accumulation such as Loury (1981), Benabou (1996), Fernandez and Rogerson (1998) or Blankenau and Simpson (2004) only allow for one public input into human capital production. Here we find it useful to disaggregate public education inputs into teachers H_t^{ge} and material inputs E_t such as textbooks, computers and buildings.

The government uses effective labor (human capital) of civil servants employed in the non-educational sector $H_t^{ui} = H_t N_t^{ui} = H_t (1-a) N_t^g$ and public capital K_t^G to produce services according to

$$G_t = Y_t^G = Z \left[\left(K_t^G\right)^{\eta_2} + \chi_2 \left(H_t^{ui}\right)^{\eta_2} \right]^{1/\eta_2}, \tag{2}$$

where $Z > 0$ and $\eta_2 \leq 1$. Public capital evolves according to

$$K_{t+1}^G = (1-\delta_G) K_t^G + I_t^G. \tag{3}$$

The government collects two kinds of labor income taxes in the public sector, the standard tax on labor income τ_{Lt}^g and an additional social security contribution rate τ_{Lt}^{ssg}. Workers in the private sector pay the tax rates τ_{Lt}^p and τ_{Lt}^{ssp}. In addition, capital income is taxed at rate τ_{Kt}. The stock of debt that the government can issue in period t is B_t. In period t the government faces the following expenditures (where we will express expenditures for government program i as fixed share $\Delta_{i,t}$ of output Y_t):

1. public education expenditures
$$E_t = \Delta_{E,t} Y_t, \tag{4}$$

2. investments in public capital
$$I_t^G = \Delta_{G,t} Y_t, \tag{5}$$

3. transfer payments to the old who were employed in the private sector
$$T_t^p = \pi_t \Psi^p w_t^p H_t N_{t-1}^p = \Delta_{T^p,t} Y_t, \tag{6}$$

4. wage payments of the current civil servants $w_t^g H_t N_t^g$,

5. pensions of last period's civil servants $\pi_t \Psi^g w_t^g H_t N_{t-1}^g$,

6. payments of public debt $\left(1 + r_t^b\right) B_t$.

Public pensions are indexed to this period's public sector wages, where $w_t^g H_t$ is an individual public employee's wage income. The total wage bill of the public sector in a given period is $w_t^g H_t N_t^g$. Since $w_t^g H_t$ is the average wage of an individual agent in a period (which is roughly 30 years long), the question arises what fraction of this current wage is paid out to retirees. In order to capture different levels of generosity of a pension system we express the amount of pensions paid to public sector retirees as

$$T_t^g = \pi_t \Psi^g w_t^g H_t N_{t-1}^g, \tag{7}$$

where $\Psi^g > 0$. If $\Psi^g \in (0,1)$ then pensions paid are only a fraction of the current average wage. The larger Ψ^g becomes the more generous the public pension system becomes. As $\Psi^g > 1$ the pensions paid are actually higher than current average wages.[1] In order to calculate the total amount of public pensions paid to retired civil servants we multiply the individual wage of a current civil servant $w_t^g H_t$ by the number of public sector retirees (the public employees of the previous period) N_{t-1}^g and by the generosity factor Ψ^g. The government budget constraint can be written as

$$\begin{aligned} & \left(1+r_t^b\right) B_t + \Delta_{E,t} Y_t + \Delta_{G,t} Y_t + \overbrace{\pi_t \Psi^p w_t^p H_t N_{t-1}^p}^{\text{private pension } T^p} + \overbrace{w_t^g H_t N_t^g}^{\text{public wages}} + \overbrace{\pi_t \Psi^g w_t^g H_t N_{t-1}^g}^{\text{public pension } T^g} \\ = \; & B_{t+1} + \left(\tau_{L,t}^{ssg} + \tau_{L,t}^g\right) w_t^g H_t N_t^g + \left(\tau_{L,t}^{ssp} + \tau_{L,t}^{sspf} + \tau_{L,t}^p\right) w_t^p H_t N_t^p \\ & + \tau_{K,t} q_t K_t + \left(1-\pi_t\right) R_t K_t, \end{aligned} \tag{8}$$

where τ_{Lt}^g and $\tau_{L,t}^p$ are labor taxes in the government and private sector respectively, $\tau_{L,t}^{ssg}$ and $\tau_{L,t}^{ssp}$ are payroll taxes for social security, $\tau_{L,t}^{sspf}$ is the employer (firm) contribution to social security in the private sector, $\tau_{K,t}$ is the capital tax, $\Delta_{E,t}$ is the fraction spent on public education, $\Delta_{G,t}$ is the fraction of GDP spent on increasing the public capital stock, $\Delta_{T,t}$ is the fraction of GDP that goes to retired private sector employees, and Ψ^g is the parameter of generosity of the public sector pension system. We assume that government behavior is exogenous.

2.2 Household Problem

We can now state the household problem as

$$\max_{c_t^j, c_{t+1}^j, i_{t+1}^j} \frac{1}{1-\sigma}\left[\left(c_t^j\right)^{1-\sigma} + \Theta G_t^{1-\sigma}\right] + (\pi\beta)\frac{1}{1-\sigma}\left[\left(c_{t+1}^j\right)^{1-\sigma} + \Theta G_{t+1}^{1-\sigma}\right] \tag{9}$$

$$s.t.$$

[1] Since wages in the data are rising with age and in the model wages are constant over the entire period, we will use values of Ψ around 1.5 to capture "integrality".

$$c_t^j + i_t^j \leq \left(1 - \tau_{Lt}^{ssj} - \tau_{Lt}^j\right) w_t^j h_t \quad (10)$$
$$c_{t+1}^j \leq R_{t+1} i_t^j + \frac{T_{t+1}^j}{N_t^j}$$

where, $j = g$ if it is a public sector worker, $j = p$ if it is a private sector worker, $i_t = k_{t+1} + b_{t+1}$ is the agent's savings in form of physical capital or government bonds, R_{t+1} is the gross rate of return on investments, and $T_{t+1}^j$is a government transfer received when old.[2] Household j takes the level of the public good G_t as well as all tax rates and prices as given.

2.3 Firm Problem

The firm's problem is standard. Note, however, that the firm takes the level of the public good as given so that the firm only chooses to hire physical capital and human capital. Note also that the government collects a social security tax from the firm at the rate τ_t^{sspf}. Thus the firm's problem is

$$\max_{\left(H_t^p, K_t\right)} F\left(G_t, K_t, H_t^p\right) - \left(1 + \tau_t^{sspf}\right) w_t^p H_t^p - q_t K_t,$$

given $\left(G_{t,}\tau_t^{sspf}, w_t^p, q_t\right)$.

2.4 Definition of Equilibrium

Given the government policy $\left\{\tau_{Lt}^p, \tau_{Lt}^g, \tau_{Lt}^{ssp}, \tau_{Lt}^{ssg}, \tau_{Lt}^{sspf}, \tau_{Kt}, \Delta_{E,t}, \Delta_{K^G,t}, \Delta_{T,t}, w_t^g, N_t^g, \Psi^g\right\}_{t=0}^{\infty}$, a competitive equilibrium is a collection of sequences of decisions of privately employed households $\left\{c_t^p, c_{t+1}^p, k_{t+1}^p, b_{t+1}^p, h_{t+1}^p\right\}_{t=0}^{\infty}$, sequences of decisions of publicly employed households $\left\{c_t^g, c_{t+1}^g, k_{t+1}^g, b_{t+1}^g, h_{t+1}^g\right\}_{t=0}^{\infty}$, sequences of aggregate stocks of private physical capital and private human capital $\{K_t, H_t^p\}_{t=0}^{\infty}$, sequences of aggregate stocks of public physical capital and public human capital $\left\{K_t^G, H_t^g\right\}_{t=0}^{\infty}$, sequences of factor prices $\left\{w_t^p, q_t, r_t^b\right\}_{t=0}^{\infty}$ such that

(i) the sequence $\left\{c_t^p, c_{t+1}^p, k_{t+1}^p, b_{t+1}^p, h_{t+1}^p\right\}_{t=0}^{\infty}$ solves the maximization problem of the privately employed household (9) with $j = p$ and the sequence $\left\{c_t^g, c_{t+1}^g, k_{t+1}^g, b_{t+1}^g, h_{t+1}^g\right\}_{t=0}^{\infty}$ solves the maximization problem of the publicly employed household (9) with $j = g$;

[2] The wage of an agent of group j is $w_t^j h_t$. We assume that human capital in the public and private sector is the same, only the fraction employed will differ, so that in the aggregate we will have $h_t = H_t$ and the fraction employed by the private sector is $H_t N_t^r$ and the fraction employed by the public sector is $H_t N_t^u$.

(ii) factor prices are determined by

$$q_t = \alpha_2 \frac{Y_t}{K_t}, \tag{11}$$

$$w_t^p = \frac{\alpha_3}{\left(1+\tau_t^{sspf}\right)} \frac{Y_t}{H_t^p} = \frac{\alpha_3}{\left(1+\tau_t^{sspf}\right)} \frac{Y_t}{\left(1-N_t^g\right) H_t}, \tag{12}$$

$$R_t = \left(1+r_t^b\right) = \left(1-\tau_t^k\right) q_t + 1 - \delta,$$

(iii) capital markets clear, so that aggregate capital stocks are given by

$$\begin{aligned} I_t &= i_t^p(1-N_t^g) + i_t^g N_t^g = K_{t+1} + B_{t+1}, \\ H_t &= H_t(1-N_t^g) + H_t N_t^g = H_t^p + H_t^g, \end{aligned}$$

(iv) commodity markets clear

$$\begin{aligned} \pi C_{t-1}^p + C_t^p + \pi C_{t-1}^g + C_t^g + K_{t+1} + I_t^G + E_t &= Y_t + (1-\delta) K_t, \\ G_t &= Y_t^G, \end{aligned}$$

(vi) and the government budget constraint (8) holds.

3 Data and Calibration

In this section we calibrate the model to the economy of Brazil. Brazil runs two separate pension systems for the public and the private sector. The public sector system is very generous. There are two constitutional provisions that guide the implementation of the public sector pension program. The requirement of "Integrality" equate pension payments to the last and highest pay check of civil servants. The provision of "Parity" indexes pensions to nominal wages paid to civil servants.

Overall, the public sector pension system accounts for 50% of all retirement payments, whereas public sector retirees only account for 5% of all retirees in Brazil. The average contribution rate of civil servants towards their pension fund is 11%. In the private sector the contribution rates are much higher, roughly 27% (7.6% employees contribution and 20% employer contribution) in the manufacturing and service sector. In the agricultural (rural) sector contribution rates are somewhat lower and range around 16%. The average pension paid to private sector retirees amounts to 70% to 80% of their wage income.[3] According to Souza et al. (2004) the deficit of the pension system amounts to roughly 4.5% of GDP, 3.5% is caused by the public sector, the remaining 1% comes from the private sector.

[3] See Bonturi (2002) for more detailed information about the Brazilian pension system.

The generosity of the public sector pension system has led to concerns about its sustainability. These concerns inspired the original bill of the Constitutional Amendment 40 (Lula Reform 2003) which had two main objectives. First, it aimed at reducing the huge deficit in the civil sector pension system. Second, it aimed at making the public system more similar to the private sector system to improve equity. The changes that were actually approved fell short of the original goals and mainly affect future public servants.[4]

Table 1 reports the preference and technology parameters. The preference parameters are perhaps non controversial. The discount factor is a standard one year estimate. Since one period is roughly 30 years long, we scale the discount factor accordingly. The parameter $\theta = 0.95$, so that the weight consumers place in their utility function on publicly provided goods is small.

Note that for the parameters of the consumption goods technology we are imposing constant returns to scale in the two private factors. Note also that capital's share of 0.4 is large relative to the estimates reported in Gollin (2002), but this relatively large parameter value is consistent with estimates for Brazil in Elias (1992) and with values used by Barro and Sala-i-Martin (2004).

The value for the elasticity of output with respect to infrastructure capital, α_1 lies between estimates by Holtz-Eakin (1994) and Ai and Cassou (1995). For the parameter η_2 in the government technology we use a value of 0.5 as a benchmark, but we will use other parameter values in our sensitivity analysis. We are not aware of any estimates of η_2. We set the parameter χ_2, which measures the labor intensity of this technology, equal to unity.

We use a value of 0.1 for the learning elasticity with respect to public expenditure. This is consistent with an estimate by Card and Krueger (1992) and values used by Fernandez and Rogerson (1996) and by Rangazas (2000). We are also not aware of any estimates of η_1. We thus use $\eta_1 = 0.5$ as a benchmark and perform sensitivity analysis using a variety of values for η_1.

[4] Souza et al. (2004) contains further details of the pension reform in Brazil.

The government budget constraint becomes

$$\overbrace{\Delta_{B,t}Y_t}^{B_{t+1}\text{ new debt}} + \overbrace{\left[\frac{\alpha_3}{1+\tau_t^{sspf}}\left(\tau_{L,t}^{g}\xi\frac{N^c}{1-N_t^g}+\tau_{L,t}^{p}\right)+\tau_{K,t}\alpha_2\right]}^{\text{tax revenue excl. ear market social sec. contribution rates (27\%)}}Y_t$$

$$= \overbrace{R_t\Delta_{B,t-1}Y_{t-1}}^{R_tB_t\text{ debt service}} + \overbrace{[\Delta_{E,t}+\Delta_{G,t}]}^{\text{Education + Investments (5\%)}} Y_t + \overbrace{\xi\frac{\alpha_3}{1+\tau_t^{sspf}}\frac{N_t^g}{1-N_t^g}Y_t}^{\text{public wages (10\%)}}$$

$$+\overbrace{\left[\overbrace{\Psi^p\frac{\alpha_3}{1+\tau_t^{sspf}}}^{\text{private pension }T^p} + \overbrace{\Psi^g\xi\frac{\alpha_3}{1+\tau_t^{sspf}}\frac{N_t^g}{1-N_t^g}}^{\text{public pension }T^g} - \overbrace{\frac{\alpha_3}{1+\tau_t^{sspf}}\left(\xi\frac{N_t^g}{1-N_t^g}\tau_{L,t}^{ssg}+\tau_{L,t}^{ssp}+\tau_{L,t}^{sspf}\right)}^{\text{contribution rate to pension}}\right]}^{\text{net pension balance (+5.5\%)}}Y_t.$$

Table 2 reports the specific public policy parameters we use for the calibration exercise. The top panel in table 2 contains data on government expenditures, the second panel contains data on tax rates, while the third panel contains data on the relative size of the public and private labor force.

We set public expenditures on education exclusive of teacher salaries equal to 1% of GDP. According to The Economist (Feb. 20, 2003), total public education expenditure in Brazil in 1999 was 5.1% of GDP. We subtract 25% which is spent on tertiary education, since only 2% of all students attend college, leaving us with 3.825% of GDP. We assume that about 75% of that is spent on salaries of teachers and administrators, leaving about 1% of GDP for buildings, computers, textbooks, etc.

According to Calderon, Easterly and Serven (2003), investment in infrastructure is about 1% of GDP. Wages to current civil servants amount to about 3.5% of GDP. According to the Ministerio de Previdencia e Assistencia Social of Brazil transfers to the old in the private sector amount to 6.6% of GDP, while public sector pensions amount to about 5% of GDP (see Souza et al. (2004)).

In our model public sector wages are higher than private sector wages by a factor ξ. We do not have data on ξ and use $\xi = 1.28$.

In order to model integrality, we need a measure of wages in the last years of one's career relative to wages averaged over the entire career. We set this number $\Psi^g = 1.5$.

Basically all of our data on tax rates come from Souza et al. (2004). The social security tax rate levied from both public sector workers is 11% of wage income. In the private sector employers add 10% of the wage bill to the pension fund.[5]

The labor income tax rate for both types of employees net of social security contributions is 9%. The capital tax rate is 9% resulting in tax revenue as a fraction of GDP of 27% ex-

[5]Since our model does not account for all government expenditure, our tax rate on employers is lower than the 20% reported by (Souza et al., 2004, p. 5).

cluding social security contribution rates. Once we include debt financing to the government budget constraint, the capital tax rate is considerably higher and reaches 53%.

According to the Social Security Ministry of Brazil in 2002 there are about 5.2 million civil servants in Brazil; this constitutes 6% out of a labor force of about 85 million. According to the Global Education Database, there are approximately 2.17 million teachers in Brazil. Thus we set $a = 42\%$.

4 Policy Experiments and Results

4.1 Public Pensions (Ψ^g) vs. Education (Δ_E)

In this experiment we use the newly available government revenue from making public pensions less generous to finance increases in public education expenditures. According to figure 1, this policy reform raises steady state income. There are two effects, a direct effect on savings and an opportunity cost effect of being able to use the released public funds for more material inputs into education. The intuition is clear: Decreasing Ψ^g increases savings by public sector workers, which in turn increases steady state capital and output. Using the extra revenue to fund higher education increases the steady state level of human capital, hence the rate of return on saving, which again increases the capital stock and steady state GDP. This effect is large. Reducing Ψ^g from 1.5 to 1.25 increases steady state GDP by more than 12%.

This policy experiment affects the allocation of funds into different technologies. The results are therefore influenced by the relative total factor productivities (TFPs) in the 3 sectors, A in final goods sector, Z in the public goods sector and D in the human capital sector. Given the paucity of estimates of TFP in multi-sector models we perform sensitivity analysis on TFP in the final goods sector A and normalize $Z = D = 1$. We report the results of this analysis in table 4. We allow A to vary from 10 to 60. The resulting steady state increase in output due to a decrease in Ψ^g from 1.5 to 1.25 varies from 10% to 24%.

In addition, the size of these effects depends upon the technology parameters, especially on the size of γ_1, the elasticity of learning output with respect to public expenditures. We summarize the results of this sensitivity analysis in table 5 where we allow γ_1 to vary from 0.05 to 0.15. The effects on steady state income from reducing Ψ^g from 1.5 to 1.25 vary from about 7% to over 21%.

Little is known in the literature on empirical education production functions and about the elasticity of substitution between teachers and material education inputs. In table 6 we illustrate how shifting public funds from public sector pensions to education depends upon η_1, the (inverse of the) elasticity of substitution in the education production function. We see from table 6 that our results are relatively sensitive to sizeable changes in η_1. As η_1 varies from 0 to 1 the effect of reducing Ψ^g from 1.5 to 1.25 varies between 6% and 21%.

4.2 Public Pensions (Ψ^g) vs. Public Investment (Δ_G)

In the second experiment the extra revenue from cutting public sector pensions is used to invest in public sector capital. The results are illustrated in figure 2. Qualitatively these results in figure 2 are the same as those in figure 1. The only difference is quantitative. For the base line parameters the effects from using the freed-up resources for public investment generates smaller effects on steady state GDP than using these resources for public education. Reducing Ψ^g from 1.5 to 1.25 causes GDP to increase by approximately 10%. The corresponding increase when these funds are used for education is more than 12%.

We again perform sensitivity analysis on TFP in the final goods sector and allow A to vary from 10 to 60. Reducing Ψ^g from 1.5 to 1.25 causes GDP to increase by 9% if $A = 10$ and 13% for $A = 60$ (see table 7). The results are less sensitive than in the case where investments into public education increase due to the pension reform as was reported in table 4.

In table 8 we show how sensitive the results are with respect to changes in α_1, the elasticity of output with respect to public capital. We allow α_1 to vary from 0.05 to 0.15. For this range of parameter values reducing Ψ^g from 1.5 to 1.25 increases steady state output by 5% and about 15%. Thus, for realistic parameter values the effects of reallocating funds to public investment can be enormous.

In table 9 we again compare how shifting public funds from public sector pensions into public sector capital depending on η_2,the elasticity of substitution in the public production function. The effects on steady state income of using the extra revenue from public sector pensions for investment in infrastructure are quite sensitive to changes in η_2. As η_2 declines, the effect on output declines as well. If $\eta_2 = -1$, reducing Ψ^g from 1.50 to 1.25 increases steady state output by "only" 2%.

4.3 Labor Tax τ^L or Capital Tax τ^K Adjust

In this policy experiment we decrease the generosity of public pensions Ψ^g and let the labor tax τ_L or the capital tax rate τ_K adjust. Figure 3 and figure 4 report the respective effects. When Ψ^g drops from 1.5 to 1.25, and τ_L adjusts downwards, then output increases by about 10%. There are several effects at work here; all effects go in the same direction.

First, there is an income effect due to the lower labor tax rate on the young. Since the young are the only savers in the model, increasing their after tax income increases savings, capital accumulation and steady state income. This effect is reinforced by a simultaneous drop in the real interest rate, which lowers debt service and allows a further reduction in the labor income tax rate. This additional reduction in the income tax rate further stimulates capital accumulation and increases steady state income. There is also an increase in savings of civil servants due to the reduction of their expected future pension payments. These effects together cause a massive effect on steady state output.[6]

An adjustment of τ_K has a smaller effect on output of roughly 2% when Ψ^g declines from 1.5 to 1.25 (see figure 4).

4.4 Transition with Logarithmic Utility Function

In order to calculate transition paths we simplify the model further and drop bonds out of the system. In addition, we assume log-utility and full depreciation of public capital. The transition generation, that is the generation that is born in the old steady state and gets surprised by the policy reform gets to keep its pension package. The new replacement rate of public pension only applies to the generation born after the policy change (grandfathering). The system then reduces to:

$$K_{t+1} = \frac{\left[\frac{N_t^g \xi}{\left(1-N_t^g\right)}+1\right]\left[\left(\frac{(\pi\beta)}{1+(\pi\beta)}\right)\frac{\left(1-\tau_{Lt}^{ssp}-\tau_{Lt}\right)\alpha_3}{\left(1+\tau_t^{sspf}\right)}\right]Y_t}{\left[1+N_t^g\frac{1}{(1+(\pi\beta))}\frac{\Psi^g\xi\alpha_3}{\left(1+\tau_{t+1}^{sspf}\right)\left(1-N_{t+1}^g\right)}\frac{1}{\left(1-\tau_{t+1}^k\right)\alpha_2}+\frac{1}{1+(\pi\beta)}\frac{\Psi^p\alpha_3}{\left(1+\tau_{t+1}^{sspf}\right)}\frac{1}{\left(1-\tau_{t+1}^k\right)\alpha_2}\right]},$$

$$H_{t+1} = D\left[\left(aN_t^g H_t\right)^{\eta_1}+\chi_1\left(\Delta_t^E Y_t\right)^{\eta_1}\right]^{\frac{\gamma_1}{\eta_1}} H_t^{\gamma_2},$$

$$G_{t+1} = Z\left[\left(\Delta_t^G Y_t\right)^{\eta_2}+\chi_2\left((1-a)\,N_{t+1}^g H_{t+1}\right)^{\eta_2}\right]^{1/\eta_2},$$

$$Y_t = AG_t^{\alpha_1}K_t^{\alpha_2}\left(\left(1-N_t^g\right)H_t\right)^{\alpha_3}$$

$$R_t = \frac{\left(1-\tau_{Kt}\right)\alpha_2}{K_t}Y_t, \text{ and}$$

[6] In addition to the steady state equilibrium depicted in figure 3 there is a second type of steady state equilibrium in which a decrease of Ψ causes the interest rate R and the labor tax rate τ_L to rise. An increase in R is then consistent with lower savings, lower investment and hence higher marginal product of capital such that the government budget constraint is still satisfied. All these together result in a decrease of steady state output.

$$
\begin{aligned}
& R_t \Delta_t^B Y_t + \Delta_t^E Y_t + \Delta_t^G Y_t + \pi_t \Psi^p \frac{\alpha_3}{\left(1+\tau_t^{sspf}\right)} Y_t \\
& + \xi \frac{\alpha_3}{\left(1+\tau_t^{sspf}\right)} \frac{Y_t}{(1-N^g)} N^g + \pi_t \Psi^g \xi \frac{\alpha_3}{\left(1+\tau_t^{sspf}\right)} \frac{Y_t}{(1-N^g)} N^g \\
= & \\
& \Delta_{t+1}^B Y_{t+1} + \left(\tau_{Lt}^{ssg} + \tau_{Lt}^g\right) \xi \frac{\alpha_3}{\left(1+\tau_t^{sspf}\right)} \frac{Y_t}{(1-N^g)} N^g \\
& + \left(\tau_{Lt}^{ssp} + \tau_{Lt}^{sspf} + \tau_{Lt}^p\right) \frac{\alpha_3}{\left(1+\tau_t^{sspf}\right)} Y_t + \tau_{Kt} \alpha_2 Y_t + (1-\pi_t) \alpha_2 Y_t .
\end{aligned}
$$

In figure (5) we decrease the generosity of public pension payments from $\Psi^g = 1.5$ to $\Psi^g = 1$ and let the expenditures into public education Δ^E adjust to clear the government budget constraint. In figure (6) we again decrease Ψ^g to one and let Δ^G, the investment into public capital adjust to clear the government budget constraint. In both cases the transition period is very long, approximately 15 periods (one period is roughly 30 years long).

4.5 Welfare Analysis

Figures 7, 8 and 9 report compensating consumption levels per age cohort to make agents indifferent between the benchmark case $\Psi^g = 1.5$ and the new regime with $\Psi^g = 1$ for the three policy experiments, that is (*i*) capital taxes adjust, (*ii*) public capital investments adjust and (*iii*) investments into public education adjust.

We first record the present value welfare levels of each cohort over the transition period for the case without a policy change. Second, we record welfare levels for each cohort when the government administers a change in the pension compensation scheme of civil servants. We then calculate the average per period compensating consumption for each generation that equalizes their respective lifetime welfare.

In all three figures we illustrate the average percentage of current value compensating consumption over current value consumption for each age cohort. We distinguish between private (circles), public (triangles) and aggregate (x's) welfare levels.

For case (*i*) and (*iii*) we see that civil servant generations that are born before the policy change benefit from it because of grandfathering (compare generation 0 in figures 7 and 8). Private sector workers of generation 0 lose from the reform. There are two effects at work here. When the policy reform is announced generation zero agents enter their second (or old age) period. Due to the higher savings of the new public cohorts, the interest rate drops, so that the savings income of old agents decreases. At the same time wages increase. Since pensions are indexed to current wages, the income pension income of private sector old agents increases. Since the replacement rate in the private sector is fairly low, the pension increase is not enough to offset the loss from savings income. Therefore, private sector workers of generation 0 lose from the pension reform. This happens when capital taxes or public capital investments adjust as a reaction to the public pension cuts.

All future private sector generations will benefit from the reform. All future public sector generations will lose from the reform.

Experiment (*ii*) is different in its welfare effect. Here generation 0 from the private and public sector gain from the reform (compare figure 9). Public sector workers gain due to grandfathering and increases in output and wages, which increases their indexed pension payments. Private sector workers also gain due to large increases in wages. These increases are a direct effect of the higher levels of investments into public education which directly raise worker productivity as described by equation (1). Since pensions are indexed to current wages, the drop in interest rates is more than compensated for by indexed pension income.

The fact that welfare results differ by the alternative use of public funds has important implications for implementing these types of social security reforms. Only when there is a majority of the currently alive that benefits from the reform, can we expect such reforms to be implemented. In our case the long run gains from such reforms can only be expected to be realized when the decreased generosity of pensions for civil servants results in higher education expenditure by the government.

5 Conclusion

In this paper we have used an overlapping generations model to assess the efficiency gains of re-allocating government funds from unproductive public pensions to productive investments into public education and infrastructure. We have calibrated the model to Brazil and provided extensive sensitivity analysis. We found that (*i*) the direct effects of pension reform through savings of civil servants are small, (*ii*) the indirect effects from reinvesting freed up resources into public education or infrastructure are large, and (*iii*) welfare gains for the generation born before the reform can only be realized when the newly available resources are diverted into public education. This last result is crucial since a living majority must be found in order to successfully implement such pension reforms.

Implementing a policy reform that severely restricts the generosity of public sector pensions is bound to run into strong political opposition since civil servants are typically well organized. While the long run costs of very generous pensions and the long run gains from pension reform are clear and well documented in the literature, it is crucial to find a way to overcome short run political opposition. Our model clearly shows that if the decrease in public sector pensions is accompanied by tax cuts there will be substantial welfare losses in the current generation. If current civil servants are grandfathered there will be welfare losses among the current old who work in the private sector. These workers will see a decrease in interest income since the increase in capital accumulation will lower the interest rate on their savings.

These welfare losses will most likely lead to political opposition and doom any attempts at meaningful reform. Our paper also clearly shows that this political opposition can be overcome if the released public funds are allocated to increase public education expenditure. If that is the case human capital and wage income rises and this increase in wage income can compensate the initial old in the private sector who are facing declining interest income because of the higher savings of civil servants. We expect such a result to hold in other countries in Latin America and beyond. This result should also hold if population growth and aging of the population is taken into consideration.

In this paper we have concentrated on three channels of how public sector pension reform might influence capital accumulation. Additional channels might be: (*i*) The generosity of public sector pensions influences workers' retirement decisions, which in turn has an effect on GDP. (*ii*) The generosity of public sector pensions relative to pensions in the private sector will influence how workers will be allocated across both sectors, which in turn will influence GDP. This would require the introduction of heterogenous agents who make idiosyncratic investment choices into their human capital. This extended framework would allow us to investigate changes in the quality of the public sector labor force, given a specific worker compensation package (wages plus pension plan).

In our model the publicly produced service was made available to all firms and households at a zero price. While this might be a useful assumption for the provision of infrastructure like roads and highways, it clearly does not cover all relevant cases. When governments produce goods like telecommunication services or electricity, they typically charge for these goods/services. Prices charged need not bear any particular relationship to marginal or average costs. This will impact the government budget constraint. We leave the exploration of these channels for future research.

References

Ai, Chunrong and Stephen P. Cassou. 1995. "A Normative Analysis of Public Capital." *Applied Economics* 27:1201–1209.

Aschauer, David A. 1998. "How Big Should the Public Capital Stock Be? The Relationship between Public Capital and Economic Growth." The Jerome Levy Economics Institute of Bard College, Public Policy Brief No. 43.

Auerbach, J. Alan and Laurence J. Kotlikoff. 1987. *Dynamic Fiscal Policy.* Cambridge University Press.

Barro, Robert J. 1990. "Government Spending in a Simple Model of Endogenous Growth." *Journal of Political Economy* 98(5):S103–S125.

Barro, Robert J. and Xavier Sala-i-Martin. 2004. *Economic Growth.* second ed. Cambridge: The MIT Press.

Benabou, Roland. 1996. "Heterogeneity, Stratification, and Growth: Macroeconomic Implications ofCommunity Structure and School Finance." *The American Economic Review* 86(3):584–609.

Blankenau, William F. and Nicole B. Simpson. 2004. "Public Education Expenditures and Growth." *Journal of Development Economics* 73:583–605.

Bonturi, Marcos. 2002. "The Brazilian Pension System: Recent Reforms and Challenges Ahead." OECD Economics Department Working Paper No. 340.

Bresser-Pereira, C. Luiz. 1990. The Perverse Macroeconomics of Debt, Deficit and Inflation in Brazil. In *Perspectives on the Pacific Basin Economy: A Comparison of Asia and LatinAmerica*, ed. Fukuchi T. and M. Kagami. Toquio: Insstitute of Developing Economies.

Calderon, Cesar, William Easterly and Luis Serven. 2003. *Latin America's Infrastructure in the Era of Macroeconomic Crisis.* Washington, D.C.: The World Bank.

Calderon, Cesar, William Easterly and Luis Serven. 2003. The Limits of Stabilization Infrastructure, Public Deficits and Growth inLatin America. In *The Limits of Stabilization Infrastructure, Public Deficits and Growth inLatin America*, ed. W. Easterly and L.Serven. Washington, DC: World Bank chapter Latin America's Infrastructure in the Era of Macroeconomic Crises.

Card, David and Alan B. Krueger. 1992. "Does School Quality Matter? Returns to Education and the Characteristicsof Public Schools in the United States." *Journal of Political Economy* 100:1–40.

Cassou, Steven P. and Kevin J. Lansing. 1998. "Optimal Fiscal Policy, Public Capital, and the Productivity Slowdown." *Journal of Economics Dynamics and Control* 22(6):911–935.

Elias, V. 1992. *Sources of Growth: A Study of Seven Latin American Economies.* ICS Press, San Francisco.

Fernandez, Raquel and Richard Rogerson. 1996. "Income Distribution, Communities and the Quality of Public Education." *Quarterly Journal of Economics* 111:135–164.

Fernandez, Raquel and Richard Rogerson. 1998. "Public Education and Income Distribution: A Dynamic Quantitative Evaluationof Education-Finance Reform." *American Economic Review* 88(4):813–833.

Ferreira, Pedro Cavalcanti and Leandro Goncalves do Nascimento. 2005. "Welfare and Growth Effects of Alternative Fiscal Rules for InfrastructureInvestment in Brazil." Manuscript, EPGE - FGV(604).

Foguel, Miguel N., Indermit Gill, Rosane Mendonca and Ricardo PaesdeBarros. 2000. "The Public-Private Wage Gap in Brazil." Discussion Paper No. 754, Instituto de Pesquisa Econômica Aplicada (IPEA).

Glomm, Gerhard and B. Ravikumar. 1994. "Public Investment in Infrastructure in a Simple Growth Model." *Journal of Economic Dynamics and Control* 18(6):1173–1187.

Glomm, Gerhard, Juergen Jung and Chung Tran. 2006. "Macroeconomic Implications of Early Retirement in the Public Sector: TheCase of Brazil." Caepr Working Papers 2006-008.

Gollin, Douglas. 2002. "Getting Income Shares Right." *Journal of Political Economy* 110(2):458–474.

Herwig, Immervoll, Horacio Levy, Jose Ricardo Nogueira and Cathal O'Donoghueand Rozane Bezerra de Siqueira. 2006. "The Impact of Brazil's Tax-Benefit System on Inequality and Poverty." IZA Discuassion Paper No. 2114.

Holtz-Eakin, Douglas. 1994. "Public Sector Capital and the Productivity Puzzle." *Review of Economics and Statistics* 76(1):12–21.

Hulten, Charles B. 1996. "Infrastructure Capital and Economic Growth: How well you use it may be moreimortant than how much you have." NBER Working Paper No. 5847.

Issler, João V. and Natalia S. Piqueira. 2000. "Estimating Relative Risk Aversion, the Discount Rate, and the Intertemporal Elasticity of Substitution in Consumption for Brazil Using Three Types of Utility Functions." *Brazilian Review of Econometrics* 20:201–239.

Loury, Glenn C. 1981. "Intergenerational Transfers and the Distribution of Earnings." *Econometrica* 49(4):843–867.

OECD. 2005. *OECD Economic Surveys, Brazil - Volume 2005 Issue 2.* OECD Publishing.
URL: *http://www.oecd.org/document/60/0,2340,*$en_2649_34571_34413308_{1111},00.html$

Palacios, Robert and Edward Whitehouse. 2006. "Civil-service Pension Schemes Around the World." Social Protection (SP) Discussion Paper No. 0602, The World Bank.

Panizza, Ugo. 2000. "The Public Sector Premium and the Gender Gap in Latin America: Evidence for the 1980s and 1990s." Working Paper 431, Inter-American Development Bank.

Panizza, Ugo. 2001. "Public Sector Wages and Bureaucratic Quality: Evidence from Latin America." *ECONOMIA* Fall:97–151.

Panizza, Ugo and Christine Zhen-Wei Qiang. 2005. "Public Private Wage Differential and Gender Gap in Latin America: Spoiled Bureaucrats and Exploited Women?" *The Journal of Socio-Economics* 34:810–833.

Rangazas, Peter. 2000. "Schooling and Economic Growth: A King-Rebelo Experiment with Human Capital." *Journal of Monetary Economics* 46:346–416.

Soriano, Leonardo and Márcio I. Nakane. 2003. "Real Balance in the Utility Function: Evidence from Brazil." Working Paper Series 68, Banco Central do Brasil.

Souza, Andre Portela, Helio Zylberstajn, Luis Eduardo Afonso and PriscillaMatia Flori. 2004. "Fiscal Impacts of Social Securty Reform in Brazil." Working Paper.

Souza-Sobrinho, Nelson F. 2004. "Structural Reforms and Economic Performance: Brazil in the 1990s." UCLA.

Turnovsky, Stephen J. and Walther H. Fisher. 1995. "The Composition of Government Expenditure and Its Consequences for MacroeconomicPerformance." *Journal of Economic Dynamics and Control* 19(4):747–786.

6 Appendix A: Solving the Model

6.1 Household Choices

We assume that the government indexes public worker wages to private worker wages as follows

$$w_t^g = \xi w_t^p. \tag{13}$$

We typically restrict ξ to be sufficient large so that we can assume that the government can directly set the fraction of the workforce N_t^g it wants to employ. Then total human capital employed by the public sector is $H_t^g = H_t N_t^g$. All other workers $(1 - N_t^g)$ will work in the private sector, that is $H_t^p = H_t N_t^p = H_t (1 - N_t^g)$. We justify this by assuming that agents would prefer to work for the government if lifetime income from working in the public sector exceeds lifetime income from working in the private sector.

Households can invest in two assets, physical capital and government issued bonds. In equilibrium both assets have to pay the same rate of return due to non-arbitrage conditions. If we denote $R_t = (1 - \tau_{Kt}) q_t + 1 - \delta$ as the after-tax return on capital investment and $R_t^b = \left(1 + r_t^b\right)$ as the net return on bonds, we get

$$(1 - \tau_{Kt}) q_t + 1 - \delta = 1 + r_t^b = R_t.$$

If we assume full depreciation $\delta = 1$, which is quite reasonable, given that the length of one period is 30 years. Then the interest rate condition becomes

$$(1 - \tau_{Kt}) q_t = 1 + r_t^b = R_{t+1}. \tag{14}$$

The Lagrangian of the problem becomes

$$\begin{aligned} L(\cdot) &= \frac{1}{1-\sigma}\left[\left(c_t^j\right)^{1-\sigma} + \Theta G_t^{1-\sigma}\right] + (\pi\beta)\frac{1}{1-\sigma}\left[\left(c_{t+1}^j\right)^{1-\sigma} + \Theta G_{t+1}^{1-\sigma}\right] + \\ & \lambda^j\left[\left(1 - \tau_{Lt}^{ssj} - \tau_{Lt}^j\right) w_t^j h_t + \frac{T_{t+1}^j}{R_{t+1} N_t^j} - c_t^j - \frac{c_{t+1}^j}{R_{t+1}}\right]. \end{aligned}$$

The optimal decision rules for savings and consumption are

$$
\begin{aligned}
i_t^j &= \frac{(\pi\beta)^{\frac{1}{\sigma}} R_{t+1}^{\frac{1}{\sigma}-1} \left(1-\tau_{Lt}^{ssj}-\tau_{Lt}^{j}\right) w_t^j h_t - \frac{T_{t+1}^j}{R_{t+1}N_t^j}}{1+(\pi\beta)^{\frac{1}{\sigma}} R_{t+1}^{\frac{1}{\sigma}-1}},\\
c_t^j &= \frac{I_t^j}{1+(\pi\beta)^{\frac{1}{\sigma}} R_{t+1}^{\frac{1}{\sigma}-1}} = \frac{1}{\mathcal{C}} I_t^j,\\
c_{t+1}^j &= \frac{(\pi\beta R_{t+1})^{\frac{1}{\sigma}}}{1+(\pi\beta)^{\frac{1}{\sigma}} R_{t+1}^{\frac{1}{\sigma}-1}} I_t^j = \frac{(\pi\beta R_{t+1})^{\frac{1}{\sigma}}}{\mathcal{C}} I_t^j,
\end{aligned}
$$

where $I_t^j = \left(1-\tau_{Lt}^{ssj}-\tau_{Lt}^{j}\right) w_t^j h_t + \frac{T_{t+1}^j}{R_{t+1}N_t^j}$ and $\mathcal{C} = 1+(\pi\beta)^{\frac{1}{\sigma}} R_{t+1}^{\frac{1}{\sigma}-1}$. We then obtain the investment decisions as

$$
\begin{aligned}
i_t^g &= \left(\frac{(\pi\beta)^{\frac{1}{\sigma}} R_{t+1}^{\frac{1}{\sigma}-1}}{1+(\pi\beta)^{\frac{1}{\sigma}} R_{t+1}^{\frac{1}{\sigma}-1}}\right) \frac{\left(1-\tau_{Lt}^{ssj}-\tau_{Lt}^{j}\right)\xi\alpha_3 Y_t}{\left(1+\tau_t^{sspf}\right)\left(1-N_t^g\right)} - \frac{1}{\left(1+(\pi\beta)^{\frac{1}{\sigma}} R_{t+1}^{\frac{1}{\sigma}-1}\right)} \frac{\Psi^g \xi\alpha_3}{\left(1+\tau_{t+1}^{sspf}\right)\left(1-N_{t+1}^g\right)} \frac{Y_{t+1}}{R_{t+1}},\\
i_t^p &= \left(\frac{(\pi\beta)^{\frac{1}{\sigma}} R_{t+1}^{\frac{1}{\sigma}-1}}{1+(\pi\beta)^{\frac{1}{\sigma}} R_{t+1}^{\frac{1}{\sigma}-1}}\right) \frac{\left(1-\tau_{Lt}^{ssj}-\tau_{Lt}^{j}\right)\alpha_3 Y_t}{\left(1+\tau_t^{sspf}\right)\left(1-N_t^g\right)} - \frac{1}{\left(1+(\pi\beta)^{\frac{1}{\sigma}} R_{t+1}^{\frac{1}{\sigma}-1}\right)} \frac{\Psi^p \alpha_3}{\left(1+\tau_{t+1}^{sspf}\right)\left(1-N_{t+1}^g\right)} \frac{Y_{t+1}}{R_{t+1}}.
\end{aligned}
$$

6.2 Aggregation

Adding private and public investment we get an expression for aggregate saving

$$
\begin{aligned}
K_{t+1}+B_{t+1} &= I_t^g + I_t^p = N^g i_t^g + \left(1-N^g\right) i_t^p\\
&= N_t^g \left[\begin{array}{c} \left(\frac{(\pi\beta)^{\frac{1}{\sigma}} R_{t+1}^{\frac{1}{\sigma}-1}}{1+(\pi\beta)^{\frac{1}{\sigma}} R_{t+1}^{\frac{1}{\sigma}-1}}\right) \frac{\left(1-\tau_{Lt}^{ssj}-\tau_{Lt}^{j}\right)\xi\alpha_3 Y_t}{\left(1+\tau_t^{sspf}\right)\left(1-N_t^g\right)} \\ -\frac{1}{\left(1+(\pi\beta)^{\frac{1}{\sigma}} R_{t+1}^{\frac{1}{\sigma}-1}\right)} \frac{\Psi\xi\alpha_3}{\left(1+\tau_{t+1}^{sspf}\right)\left(1-N_{t+1}^g\right)} \frac{Y_{t+1}}{R_{t+1}} \end{array}\right] +\\
&\quad \left(1-N_t^g\right) \left[\begin{array}{c} \frac{(\pi\beta)^{\frac{1}{\sigma}} R_{t+1}^{\frac{1}{\sigma}-1}}{1+(\pi\beta)^{\frac{1}{\sigma}} R_{t+1}^{\frac{1}{\sigma}-1}} \left(1-\tau_{Lt}^{ssj}-\tau_{Lt}^{j}\right) \frac{\alpha_3}{\left(1+\tau_t^{sspf}\right)} \frac{Y_t}{\left(1-N_t^g\right)} \\ -\frac{1}{1+(\pi\beta)^{\frac{1}{\sigma}} R_{t+1}^{\frac{1}{\sigma}-1}} \frac{\Psi^p \alpha_3}{\left(1+\tau_{t+1}^{sspf}\right)\left(1-N_{t+1}^g\right)} \frac{Y_{t+1}}{R_{t+1}} \end{array}\right].
\end{aligned}
$$

6.3 Steady State Equilibrium

Imposing steady state we use expression $R = \alpha_2 \left(1-\tau_K\right) \frac{Y}{K}$ from the firm's first order condition (11) and in addition we restrict government debt to be a fixed fraction of GDP so that $B = \Delta_B Y$. We then obtain

$$
\begin{aligned}
& \alpha_2 (1-\tau_K) \frac{Y}{R} + \Delta_B Y \\
= \quad & N^g \left[\left(\frac{(\pi\beta)^{\frac{1}{\sigma}} R^{\frac{1}{\sigma}-1}}{1+(\pi\beta)^{\frac{1}{\sigma}} R^{\frac{1}{\sigma}-1}} \right) \frac{(1-\tau_L^{ssg}-\tau_L^g)\,\xi\alpha_3 Y}{(1+\tau^{sspf})(1-N^g)} - \frac{1}{\left(1+(\pi\beta)^{\frac{1}{\sigma}} R^{\frac{1}{\sigma}-1}\right)} \frac{\Psi\xi\alpha_3}{(1+\tau^{sspf})(1-N^g)} \frac{Y}{R} \right] + \\
& \left[\frac{(\pi\beta)^{\frac{1}{\sigma}} R^{\frac{1}{\sigma}-1}}{1+(\pi\beta)^{\frac{1}{\sigma}} R^{\frac{1}{\sigma}-1}} (1-\tau_L^{ssp}-\tau_L^p) \frac{\alpha_3 Y}{(1+\tau^{sspf})} - \frac{1}{1+(\pi\beta)^{\frac{1}{\sigma}} R^{\frac{1}{\sigma}-1}} \frac{\Psi^p \alpha_3}{\left(1+\tau_{t+1}^{sspf}\right)} \frac{Y}{R} \right]. \qquad (15)
\end{aligned}
$$

From (1) we get an expression for output in terms of human capital

$$h_{t+1} = D\left[(H_t^{ge})^{\eta_1} + \chi_1 E_t^{\eta_1}\right]^{\frac{\gamma_1}{\eta_1}} h_t^{\gamma_2}.$$

In the steady state this becomes

$$H^{1-\gamma_2} = \left[D\left[(aN^g H)^{\eta_1} + \chi_1 (\Delta_E Y)^{\eta_1}\right]^{\frac{\gamma_1}{\eta_1}} \right]. \qquad (16)$$

Given R, we have

$$K = \frac{(1-\tau_K)\,\alpha_2}{R} Y. \qquad (17)$$

Since at steady state $K_{t+1}^G = K_t^G = K^G$, and using (5) in the law of motion for capital (3) we have

$$K^G = \frac{\Delta_G}{\delta_G} Y. \qquad (18)$$

We use (18) in the production function for the public good (2) and get

$$G = Z\left[\left(\frac{\Delta_G}{\delta_G} Y \right)^{\eta_2} + \chi_2 \left[(1-a)\,N^g H\right]^{\eta_2} \right]^{1/\eta_2}, \qquad (19)$$

that expresses the output of the public good G as a function of human capital H. Then the steady state output is given by

$$Y = AG^{\alpha_1} K^{\alpha_2} \left[H(1-N^g)\right]^{\alpha_3}. \qquad (20)$$

The government budget constraint is

$$\begin{aligned}
& R\Delta_B + \Delta_E + \Delta_G + \mu\Psi^p \frac{\alpha_3}{(1+\tau^{sspf})} \\
& +\xi \frac{\alpha_3}{(1+\tau^{sspf})} \frac{1}{(1-N^g)} N^g + \mu\Psi^g \xi \frac{\alpha_3}{(1+\tau^{sspf})} \frac{1}{(1-N^g)} N^g \\
= & \\
& \Delta_B + \left(\tau_L^{ssg} + \tau_L^g\right) \xi \frac{\alpha_3}{(1+\tau^{sspf})} \frac{1}{(1-N^g)} N^g \\
& + \left(\tau_L^{ssp} + \tau_L^{sspf} + \tau_L^p\right) \frac{\alpha_3}{(1+\tau^{sspf})} + \tau_K \alpha_2 + (1-\pi) RK.
\end{aligned} \tag{21}$$

Since we picked government debt to be exogenous, we need to specify a new endogenous variable out of the set of government policy variables which adjusts in the policy reform to clear the budget constraint. We pick either capital tax τ_K or labor tax τ_L. Equations $(15), (16), (17), (19), (20)$ and (21) determine the steady state variables $\tau_K, (\tau_L), K, R, Y, H$ and G.

7 Appendix B: Tables and Graphs

Parameters		
Preferences		
Inverse of Intertemporal Elasticity of Substitution	$\sigma = 1.5$	Issler and Piqueira (2000) Soriano and Nakane (2003)
	$\Theta = 0.05$	
	$\theta = 0.95$	
Discount factor	$\beta = 0.995^{30}$	
	$\pi = 1$	
Technology		
Consumption Good:		
	$A = 15$	Lower bound of convergence of all cases incl. transitions.
	$\alpha_1 = 0.1$	Hulten (1996)
	$\alpha_2 = 0.4$	Ferreira and do Nascimento (2005)
	$\alpha_3 = 0.6$	
	$\delta = 1$	
Public Good:		
	$Z = 1$	Normalization
	$\chi_2 = 1$	Normalization
public capital and labor are substitutes:	$\eta_2 = 0.5$	
public capital and labor are complements:	$\eta_2 = -0.25$	
	$\delta_G = 0.8$	
Human Capital:		
	$D = 1$	
	$\chi_1 = 0.2$	
	$\eta_1 = 0.5$	
	$\gamma_1 = 0.1$	
	$\gamma_2 = 0.5$	

Table 1: Preference and Technology Parameters

	Variables for Benchmark Case:		Source
Policies:			
Δ_G	Investment in public good (in % of private sector output)	2.5%	Calderon and Serven (2003)
Δ_E	Public Education: Teacher's Salary (in % of private sector output)	1%	
Δ_{Cg}	Government residual expenditure (in % of private sector output)	3%	Social Security Ministry of Brazil (2002) and authors' calculation
Δ_B	Debt level	3%	
ξ	Public wages as a fraction of private wages	1.3	Foguel et al. (2000)
Ψ^p	Indexation parameter (generosity of private pensions)	0.14	Based on Bonturi (2002)
Ψ^g	Indexation parameter (generosity of public pensions)	1.5	Integrality
Taxes:			
$\tau_L^p = \tau_L^g$	Labor tax rate (net of social security)	20%	Ferreira and do Nascimento (2005)
τ_K	capital tax rate,with bonds	21%	Ferreira and do Nascimento (2005) reports 16%
τ_L^{ssg}	social security contribution rate of civil servants	11%	
τ_L^{ssp}	social security contribution rate of private sector employees	11%	
τ_L^{sspf}	social security contribution rate of private sector employers	10%	
Labor:			
N^g	fraction of civil servants	6%	Social Security Ministry of Brazil (2002)
N^p	private sector employees	94%	
a	fraction of teachers in public sector	42%	

Table 2: Government Policy Parameters

.

	Variables for Benchmark		Source
$\frac{K}{Y}$	Capital output ratio	3	Bresser-Pereira (1990) and Souza-Sobrinho (2004)
$\frac{K_g}{K}$	Public capital to private capital ratio	32%	Aschauer (1998) reports 44.6% for the U.S.
Gov't Size:			
	Tax revenue (in % of private sector output)	32.21%	Herwig et al. (2006) report 35% of GDP.
N^g	Endogenous fraction of civil servants per cohort with early retirement	6%	Souza et al. (2004) report 6%.
	Expenditures:		
$\frac{\xi w H N^g}{Y}$	Wage bill public sector workers (in % of private sector output)	4.35%	Social Security Ministry of Brazil (2002) and authors' calculation
$\frac{\Psi^g \xi w H N^g}{Y}$	Public pensions (in % of private sector output)	6.79%	Souza et al. (2004) report 5% of GDP.
$\frac{\Psi^p w H N^p}{Y}$	Private pensions (in % of private sector output)	7.64%	Souza et al. (2004) report 6.6% of GDP.

Table 3: Model Outcomes that Match Brazilian Data

	Ψ	1	1.25	1.5	1.6
	10.000	118.356	110.156	100.000	94.279
	15.000	123.011	112.844	100.000	92.594
	20.000	126.734	115.010	100.000	91.206
	25.000	129.782	116.794	100.000	90.042
	30.000	132.325	118.288	100.000	89.051
A	35.000	134.477	119.558	100.000	88.197
	40.000	136.322	120.650	100.000	87.454
	45.000	137.921	121.599	100.000	86.802
	50.000	139.319	122.431	100.000	86.226
	55.000	140.551	123.166	100.000	85.712
	60.000	141.646	123.820	100.000	85.251

Table 4: Change in Output with Δ_E adjusting

	Ψ	1	1.25	1.5	1.6
	0.050	112.044	106.682	100.000	96.176
	0.060	113.978	107.789	100.000	95.503
	0.070	116.031	108.954	100.000	94.809
	0.080	118.213	110.181	100.000	94.093
	0.090	120.535	111.476	100.000	93.355
γ_1	0.100	123.011	112.844	100.000	92.594
	0.110	125.656	114.291	100.000	91.808
	0.120	128.485	115.824	100.000	90.998
	0.130	131.519	117.449	100.000	90.161
	0.140	134.777	119.176	100.000	89.298
	0.150	138.283	121.012	100.000	88.406

Table 5: Change in Output with Δ_E adjusting ($\eta_2 = 0.5$)

	Ψ	1	1.25	1.5	1.6
	0.000	111.110	106.472	100.000	95.375
	0.250	116.085	109.193	100.000	94.100
η_1	0.500	123.011	112.844	100.000	92.594
	0.750	131.247	117.203	100.000	90.864
	1.000	139.310	121.662	100.000	88.994

Table 6: Change in Output with Δ_E adjusting

	Ψ	1	1.25	1.5	1.6
	10.000	116.785	108.985	100.000	95.798
	15.000	118.651	110.022	100.000	95.271
	20.000	119.910	110.721	100.000	94.916
	25.000	120.837	111.234	100.000	94.656
	30.000	121.558	111.631	100.000	94.456
A	35.000	122.137	111.950	100.000	94.296
	40.000	122.616	112.213	100.000	94.164
	45.000	123.019	112.434	100.000	94.054
	50.000	123.363	112.623	100.000	93.959
	55.000	123.662	112.786	100.000	93.878
	60.000	123.923	112.929	100.000	93.807

Table 7: Change in Output with Δ_G adjusting

	Ψ	1	1.25	1.5	1.6
	0.050	110.316	105.526	100.000	97.393
	0.060	111.835	106.359	100.000	96.987
	0.070	113.424	107.224	100.000	96.571
	0.080	115.086	108.121	100.000	96.147
	0.090	116.827	109.053	100.000	95.713
α_1	0.100	118.651	110.022	100.000	95.271
	0.110	120.563	111.030	100.000	94.819
	0.120	122.571	112.077	100.000	94.358
	0.130	124.680	113.168	100.000	93.888
	0.140	126.897	114.303	100.000	93.408
	0.150	129.231	115.486	100.000	92.919

Table 8: Change in Output with Δ_G adjusting ($\eta_2 = 0.5$)

	Ψ	1	1.25	1.5	1.6
	-1.000	104.762	102.539	100.000	98.731
	-0.750	105.653	103.077	100.000	98.417
	-0.500	107.122	103.923	100.000	97.974
η_2	-0.250	109.310	105.124	100.000	97.405
	0.000	111.850	106.447	100.000	96.843
	0.250	115.984	108.616	100.000	95.887
	0.500	118.651	110.022	100.000	95.271

Table 9: Change in Output with Δ_G adjusting

	Ψ^g	1	1.25	1.5	1.6
	0.050	-4.838	-2.630	-0.004	0.743
	0.060	-3.332	-1.696	-0.002	0.078
	0.070	-1.736	-0.712	0.000	-0.608
	0.080	-0.041	0.325	0.003	-1.317
	0.090	1.762	1.418	0.005	-2.048
γ_1	0.100	3.683	2.573	0.008	-2.802
	0.110	5.732	3.794	0.010	-3.582
	0.120	7.924	5.087	0.013	-4.386
	0.130	10.270	6.458	0.016	-5.218
	0.140	12.789	7.914	0.019	-6.077
	0.150	15.497	9.463	0.022	-6.965

Table 10: Relative Difference: $\frac{\Delta_E - \Delta_G}{\Delta_G}100$, ($\eta_2 = 0.5$)

	Ψ^g	1	1.25	1.5	1.6
	0.050	9.794	6.014	0.014	-4.442
	0.060	8.626	5.360	0.013	-4.135
	0.070	7.431	4.689	0.012	-3.818
	0.080	6.209	4.002	0.010	-3.490
	0.090	4.960	3.296	0.009	-3.152
α_1	0.100	3.683	2.573	0.008	-2.802
	0.110	2.377	1.830	0.006	-2.441
	0.120	1.042	1.068	0.005	-2.068
	0.130	-0.323	0.286	0.003	-1.682
	0.140	-1.719	-0.516	0.002	-1.284
	0.150	-3.146	-1.339	0.000	-0.873

Table 11: Relative Difference: $\frac{\Delta_E - \Delta_G}{\Delta_G}100$, $(\eta_2 = 0.5)$

	Ψ^g	1	1.25	1.5	1.6
	0.050	0.710	0.380	0.001	-0.440
	0.060	1.933	1.107	0.003	-0.915
	0.070	3.220	1.868	0.004	-1.404
	0.080	4.577	2.664	0.006	-1.908
	0.090	6.008	3.500	0.008	-2.428
γ_1	0.100	7.520	4.376	0.010	-2.965
	0.110	9.119	5.296	0.012	-3.518
	0.120	10.812	6.263	0.014	-4.090
	0.130	12.608	7.281	0.016	-4.680
	0.140	14.514	8.353	0.018	-5.289
	0.150	16.541	9.483	0.020	-5.918

Table 12: Relative Difference: $\frac{\Delta_E - \Delta_G}{\Delta_G}100$, $(\eta_2 = -0.25)$

	Ψ^g	1	1.25	1.5	1.6
	0.050	11.817	6.967	0.015	-4.558
	0.060	11.026	6.491	0.014	-4.265
	0.070	10.201	5.993	0.013	-3.960
	0.080	9.342	5.475	0.012	-3.641
	0.090	8.448	4.936	0.011	-3.310
α_1	0.100	7.520	4.376	0.010	-2.965
	0.110	6.557	3.794	0.008	-2.605
	0.120	5.559	3.190	0.007	-2.231
	0.130	4.525	2.563	0.006	-1.842
	0.140	3.457	1.914	0.004	-1.437
	0.150	2.352	1.241	0.003	-1.015

Table 13: Relative Difference: $\frac{\Delta_E - \Delta_G}{\Delta_G}100$, $(\eta_2 = -0.25)$

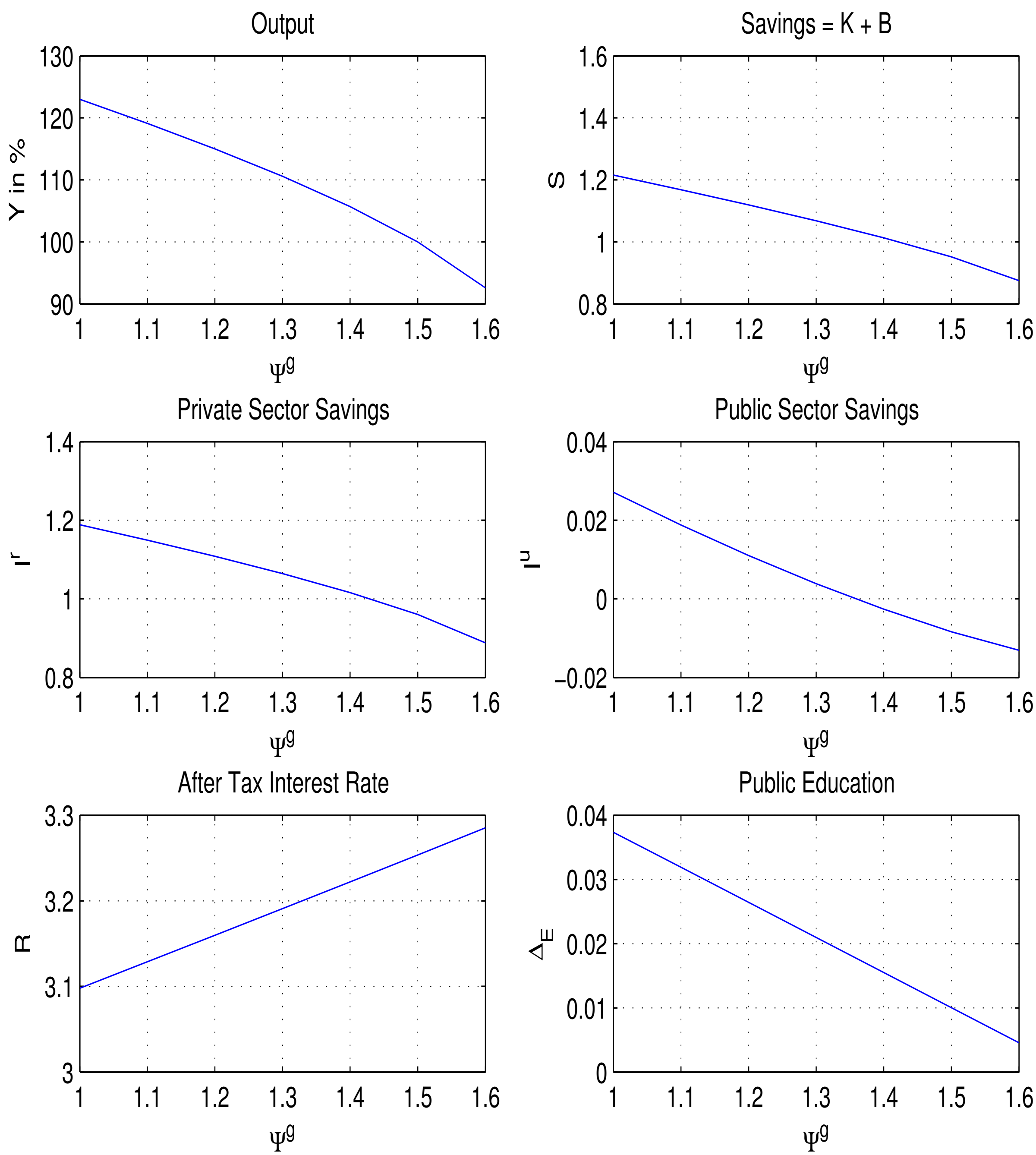

Figure 1: Effect of decreasing public sector pensions Ψ and increasing public education Δ_E

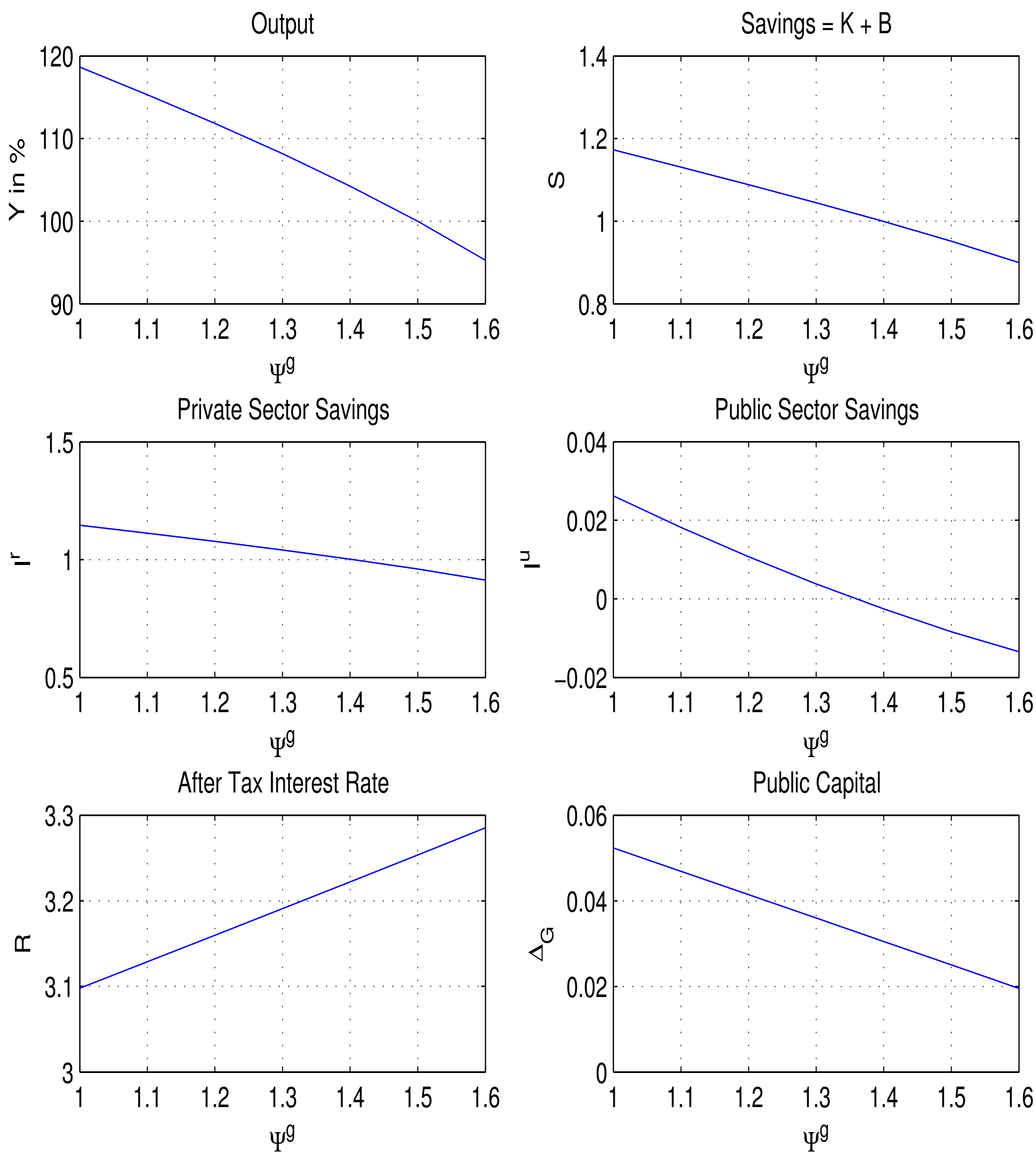

Figure 2: Effect of decreasing public sector pensions Ψ and increasing public capital Δ_G

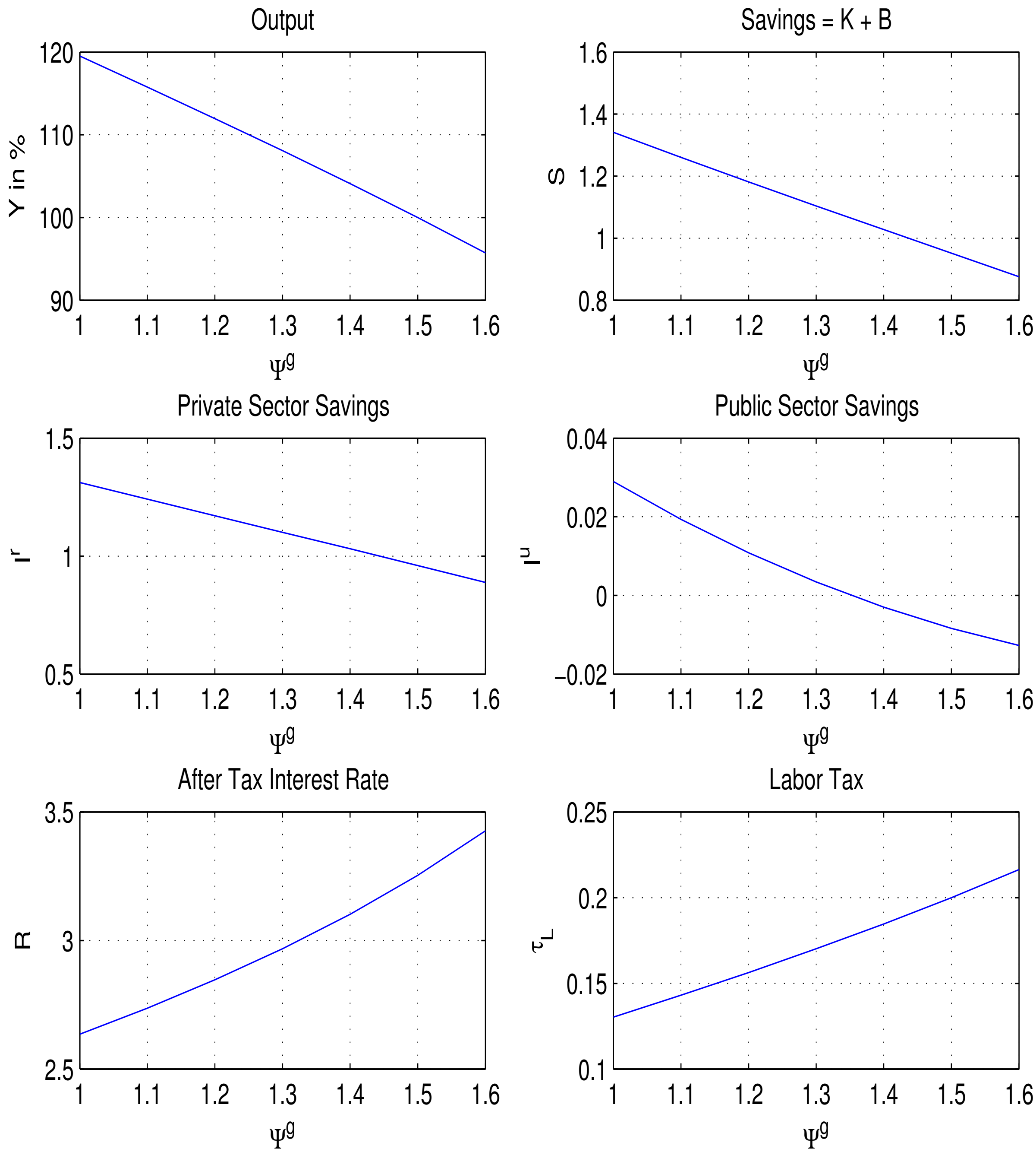

Figure 3: Effect of decreasing public sector pensions Ψ and decreasing labor taxes τ_L

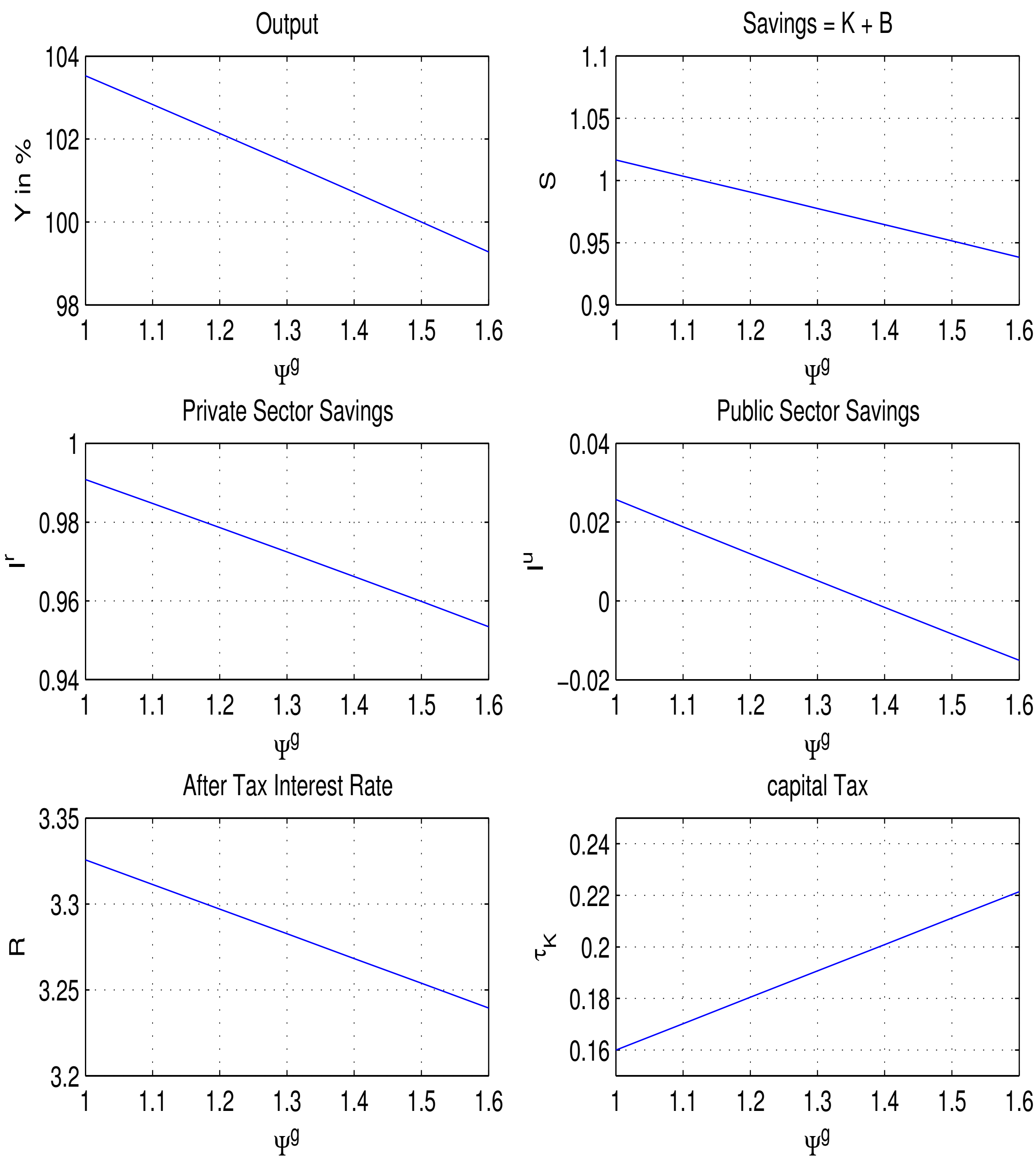

Figure 4: Effect of decreasing public sector pensions Ψ and adjusting capital taxes τ_K

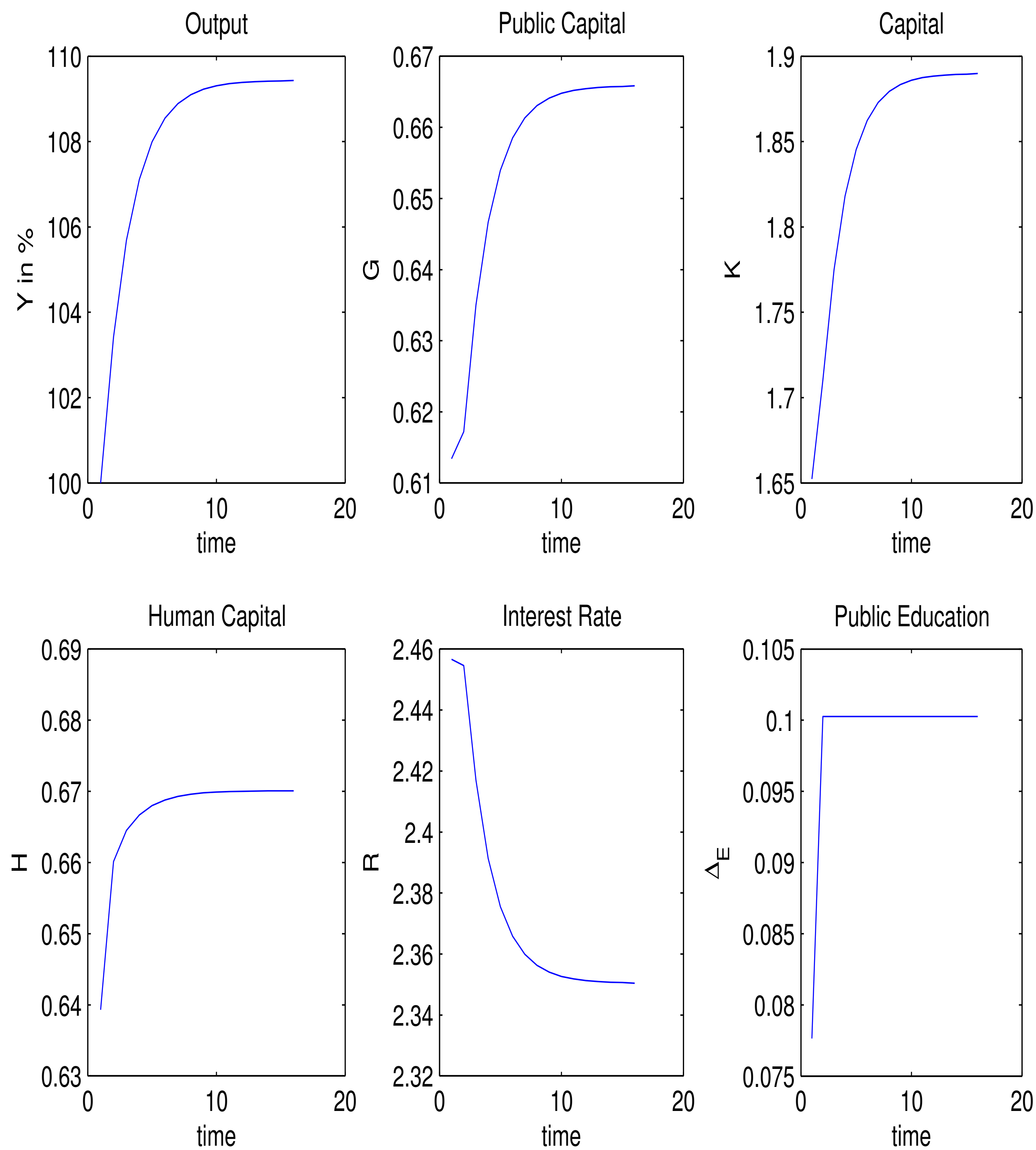

Figure 5: Transition effect of decreasing public sector pensions Ψ and adjusting public education expenditure Δ_E

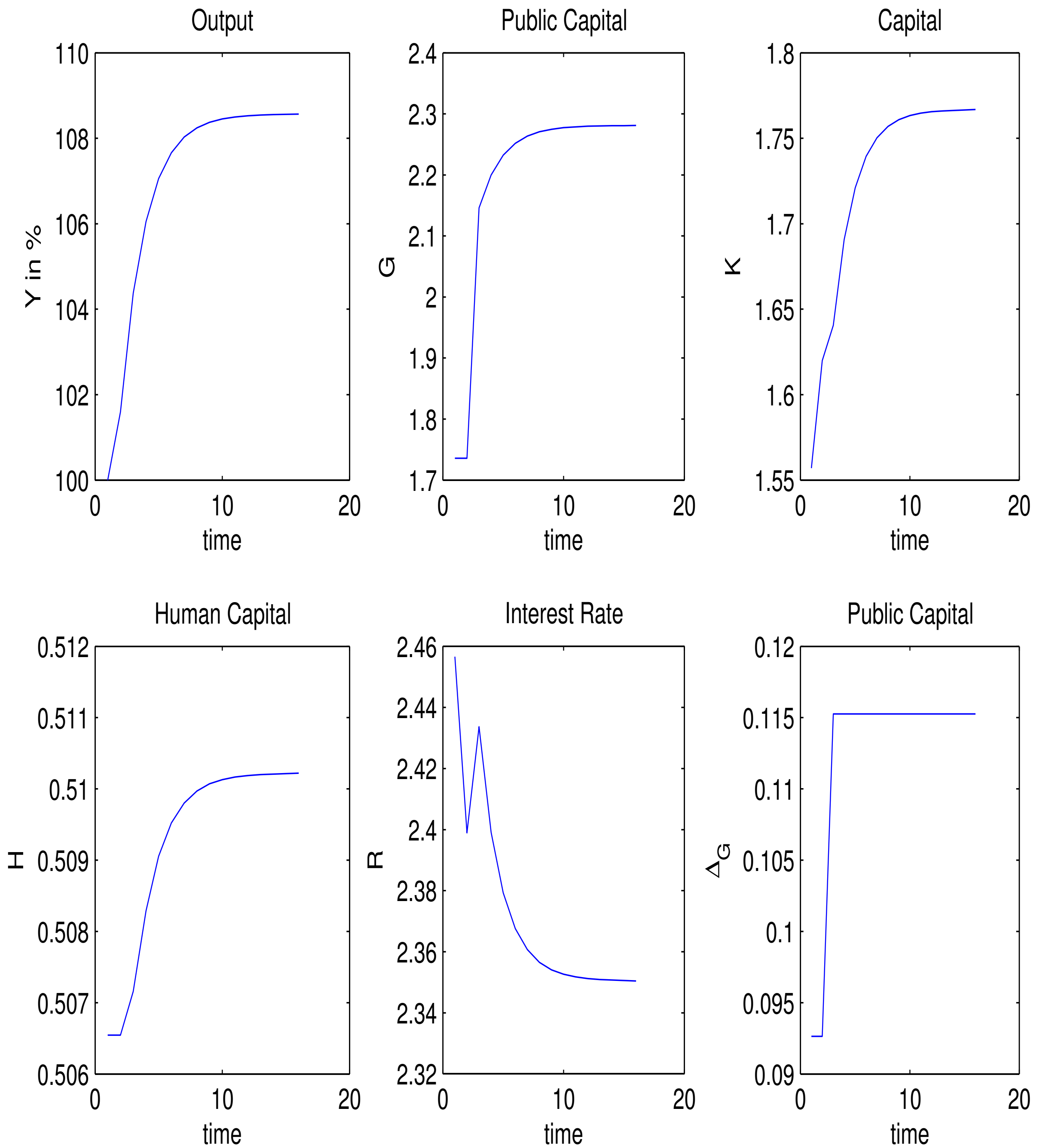

Figure 6: Transition effect of decreasing public sector pensions Ψ and adjusting public capital investment Δ_G

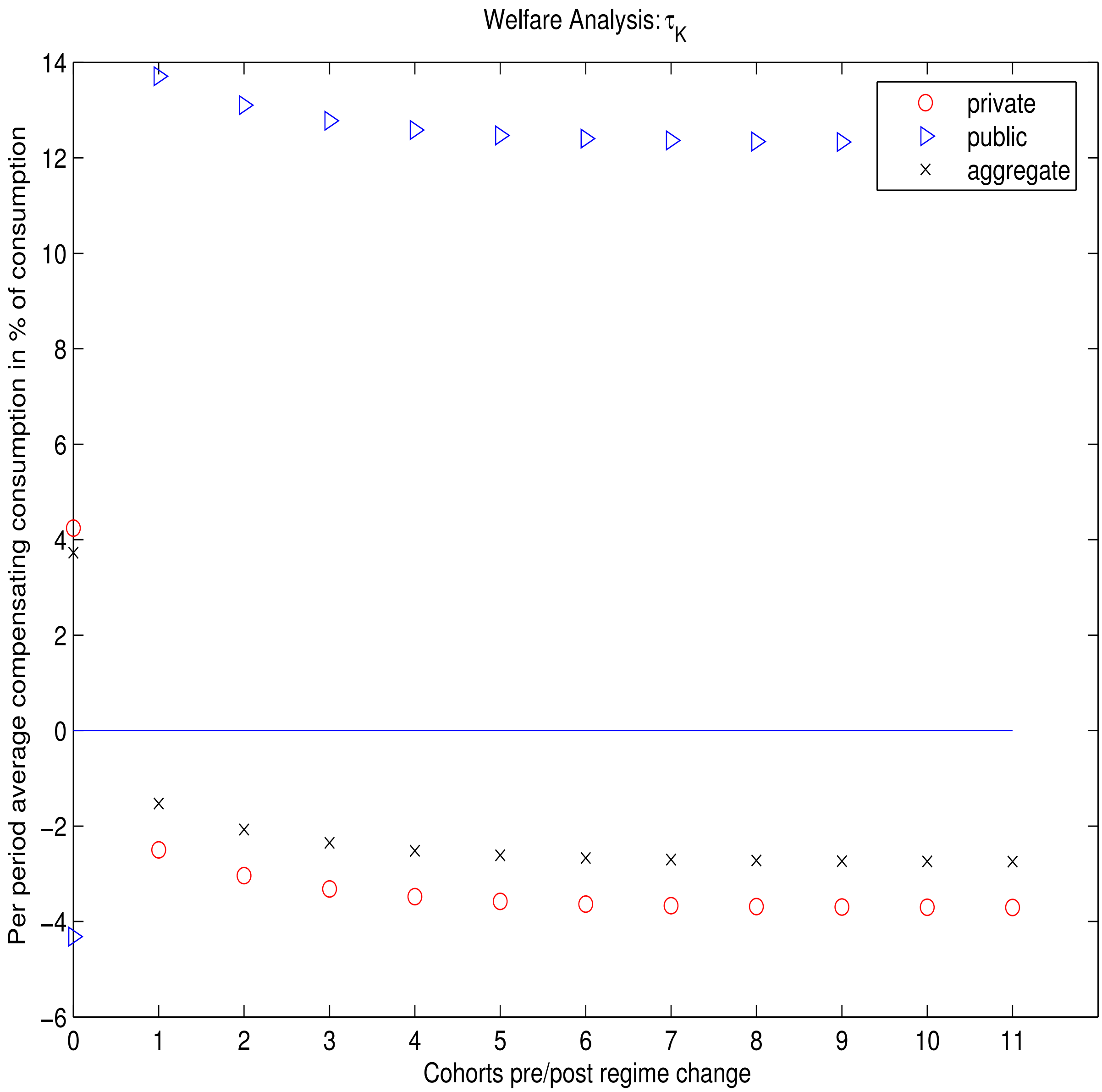

Figure 7: Compensating consumption given to individuals to offset the policy change that reduces the generosity of public pension replacement rate $Psi^g = 1.5$ to $\Psi^g = 1.0$ letting capital tax τ_K adjust to clear the government budget constraint. Compensating consumption is expressed as the average percentage of current value per period compensating consumption over current value consumption.

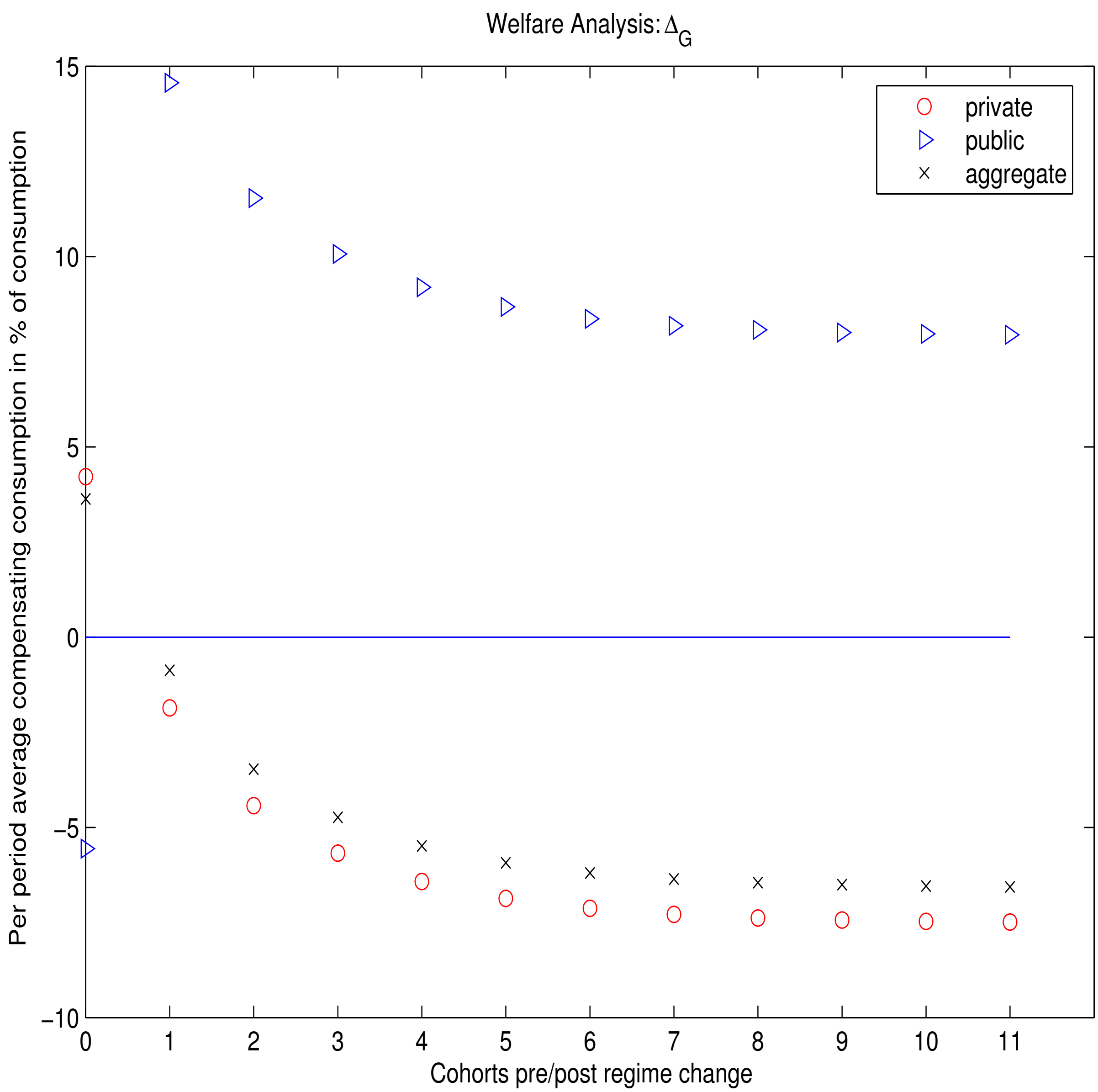

Figure 8: Compensating consumption given to individuals to offset the policy change that reduces the generosity of public pension replacement rate $Psi^g = 1.5$ to $\Psi^g = 1.0$ letting investments into public capital Δ_G adjust to clear the government budget constraint. Compensating consumption is expressed as the average percentage of current value per period compensating consumption over current value consumption.

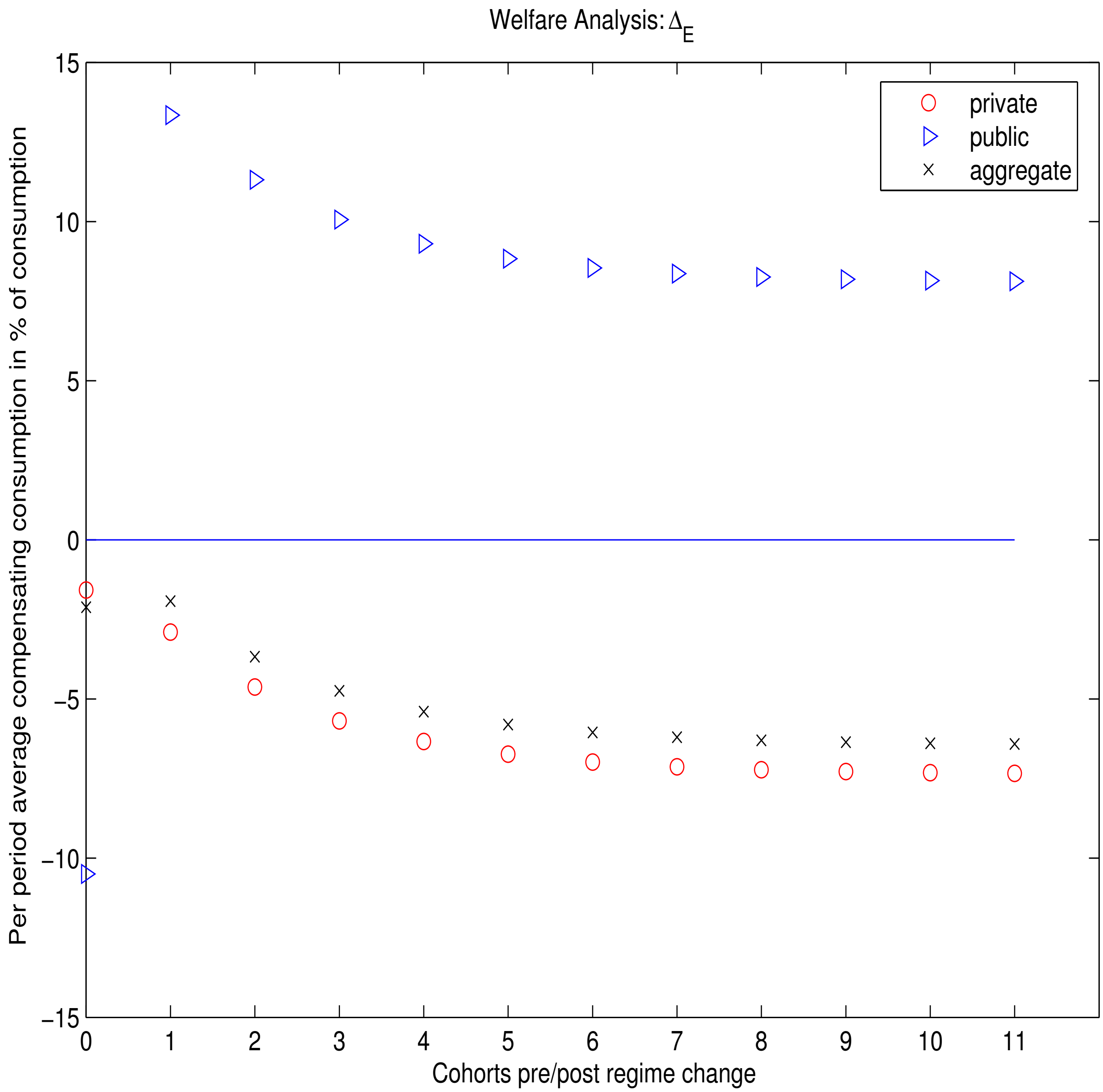

Figure 9: Compensating consumption given to individuals to offset the policy change that reduces the generosity of public pension replacement rate $Psi^g = 1.5$ to $\Psi^g = 1.0$ letting public education expenditures Δ_E adjust to clear the government budget constraint. Compensating consumption is expressed as the average percentage of current value per period compensating consumption over current value consumption.